Algebraic topology and arithmetic

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Motivation: cohomology theories and their operations

Generalized cohomology theory $\{h^n\}$: $\mathrm{Spaces} \to \mathrm{AbGroups}$

Cup product $\rightsquigarrow h^*(X)$ a graded commutative algebra over $h^*(\mathrm{pt})$

Cohomology operation Q^i: $h^*(-) \to h^{*+i}(-)$

Example (ordinary cohomology with $\mathbb{Z}/2$-coefficients)

Steenrod squares Sq^i: $H^*(-;\mathbb{Z}/2) \to H^{*+i}(-;\mathbb{Z}/2)$

Power operation $\mathrm{Sq}^i(x) = x^2$ if $i = |x|$

Steenrod algebra

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Cartan formula $\mathrm{Sq}^i(xy) = \sum_{k=0}^{i} \mathrm{Sq}^{i-k}(x)\,\mathrm{Sq}^k(y)$

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Example (complex K-theory)

Adams operations ψ^i: $K(-) \to K(-)$

Power operation $\psi^p(x) \equiv x^p \mod p$

$$\psi^i \psi^j = \psi^{ij} \qquad \psi^i(xy) = \psi^i(x)\psi^i(y)$$

J. F. Adams, *Vector fields on spheres*, Ann. of Math. (2) **75** (1962)

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Example (more – a sample)

Voevodsky, *Reduced power operations in motivic cohomology*, 2003.

Lipshitz and Sarkar, *A Steenrod square on Khovanov homology*, 2014.

Feng, *Étale Steenrod operations and the Artin–Tate pairing*, 2018.

Seidel, *Formal groups and quantum cohomology*, 2019.

Background: Chromatic Homotopy Theory 色展同伦论

A connection between Topology and Arithmetic (Quillen '69)

$$\begin{array}{ccc} \text{stable homotopy theory} & \leftrightsquigarrow & \text{1-dim formal group laws} \\ \text{complex-oriented } h^*(-) & & F(x,y) \text{ over } h^*(\mathrm{pt}) \\ c_1(L_1 \otimes L_2) & = & F\big(c_1(L_1), c_1(L_2)\big) \end{array}$$

Example

$$H^*(-;\mathbb{Z}) \leftrightsquigarrow \mathbb{G}_a(x,y) = x + y$$

$$K^*(-) \leftrightsquigarrow \mathbb{G}_m(x,y) = x + y - xy = 1 - (1-x)(1-y)$$

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Elliptic cohomology and Morava E-theory

Definition (Ando–Hopkins–Strickland '01, Lurie '09, '18)

$$\text{elliptic cohomology theory} = \left\{ \begin{array}{ll} E, & C_{E^0(\mathrm{pt})}, \\ \alpha\colon & \operatorname{Spf} E^0(\mathbb{CP}^\infty) \xrightarrow{\sim} \widehat{C} \end{array} \right\}$$

Theorem (Morava '78, Goerss–Hopkins–Miller '90s–'04)

$$\mathcal{E}\colon \{\text{formal groups over perfect fields, isos}\} \to \{\mathbb{E}_\infty\text{-ring spectra}\}$$

- $\operatorname{Spf} E^0(\mathbb{CP}^\infty) =$ the univ deformation of a fg F of height n over a perfect field k of char p
- $\pi_* E \cong \mathbb{W}(k)[\![u_1, \dots, u_{n-1}]\!][u^{\pm 1}], \quad |u| = -2$

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Power operations for Morava E-theory

$M = E$-module $\qquad \pi_0 M = [S, M]_S \cong [E, M]_E$

$$\mathbb{P}_E(M) = \bigvee_{i \geq 0} \mathbb{P}_E^i(M) = \bigvee_{i \geq 0} (\underbrace{M \wedge_E \cdots \wedge_E M}_{i\text{-fold}})_{h\Sigma_i}$$

$A =$ commutative E-algebra
$=$ algebra for the monad $\mathbb{P}_E$ with $\mu \colon \mathbb{P}_E(A) \to A$

$$\left.\begin{array}{l} \text{total power operation } \psi^i \colon \pi_0 A \to \pi_0\left(A^{B\Sigma_i^+}\right) \\ \forall \eta \in \pi_0 \mathbb{P}_E^i(E), \text{individual po } Q_\eta \colon \pi_0 A \to \pi_0 A \end{array}\right\} \overset{/I}{\rightsquigarrow} \text{additive}$$

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$$\mathbb{P}_E(M) = \bigvee_{i \geq 0} \mathbb{P}_E^i(M) = \bigvee_{i \geq 0} (\underbrace{M \wedge_E \cdots \wedge_E M}_{i\text{-fold}})_{h\Sigma_i}$$

$A =$ commutative E-algebra
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total power operation $\psi^i \colon \pi_0 A \to \pi_0\left(A^{B\Sigma_i^+}\right)$
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Power operations for Morava E-theory (height n prime p)

Theorem (Rezk '09, Barthel–Frankland '13)

If $A = K(n)$-local commutative E-algebra, then

$$\pi_* A = \text{graded amplified } L\text{-complete } \Gamma\text{-ring}$$

- $\Gamma =$ twisted bialgebra over E_0 (Dyer–Lashof algebra)
- $\exists\, Q_0 \in \Gamma$ with $Q_0(x) \equiv x^p \bmod p$ (Frobenius congruence)

Goal make this structure explicit just as for Dyer–Lashof/Steenrod operations in ordinary homology.
The case of $n = 2$ has been worked out. $\rightsquigarrow$ Arithmetic

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Power operations for Morava E-theory (height $n = 2$)

Theorem (Z. '19)

Given any Morava E-theory E of height 2 at a prime p, there is an explicit presentation for its algebra of power operations, in terms of generators $Q_i\colon\ E^0(-) \to E^0(-)$, $0 \leq i \leq p$, and quadratic relations

$$Q_i Q_0 = -\sum_{k=1}^{p-i} w_0^k\, Q_{i+k} Q_k \ - \sum_{k=1}^{p} \sum_{m=0}^{k-1} w_0^m\, d_{i,\,k-m}\, Q_m Q_k$$

for $1 \leq i \leq p$, where the coefficients w_0 and $d_{i,\,k-m}$ arise from certain modular equations for elliptic curves.

Remark The first example, for $p = 2$, was calculated by Rezk '08. These have been applied to computations in unstable v_2-periodic homotopy theory (Z. '18 and ongoing joint work with G. Wang).

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Moduli of formal groups and algebras of power operations

Recall E-theory at height n and prime p has an underlying model

$$F_k \xleftarrow{\text{univ defo}} \Gamma_{\mathbb{W}(k)[\![u_1,\ldots,u_{n-1}]\!]} \quad \leftrightsquigarrow \quad E$$

$\Gamma_{\mathbb{W}(k)[\![u_1,\ldots,u_{n-1}]\!]}$ and E each carry a self-action.

Frobenius isogenies — power operations

An equivalence of cats (Ando–Hopkins–Strickland '04, Rezk '09)

$$\left\{\begin{array}{c}\text{qcoh sheaves of grd comm algs}\\ \text{over the moduli problem of}\\ \text{defos of } F/k \text{ and Frob isogs}\end{array}\right\} \cong \left\{\begin{array}{c}\text{grd comm algs over}\\ \text{the Dyer–Lashof algebra}\\ \text{for } E\end{array}\right\}$$

Goal Compute one side explicitly to get the other side.

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Goal Compute one side explicitly to get the other side.

Moduli of formal groups and algebras of power operations

Recall E-theory at height n and prime p has an underlying model

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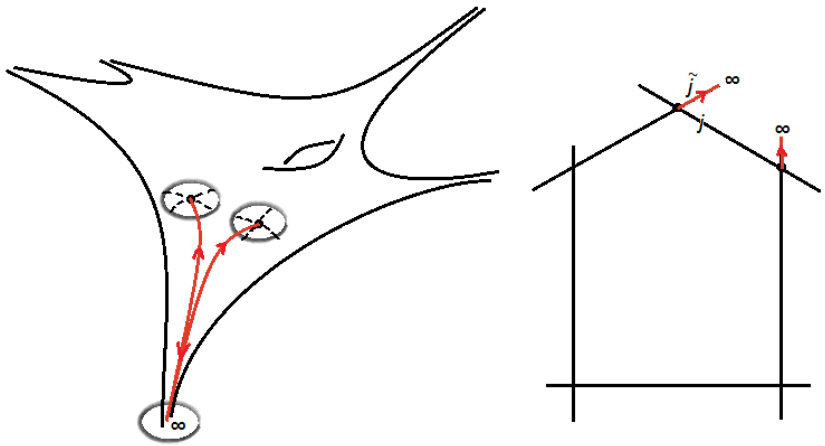

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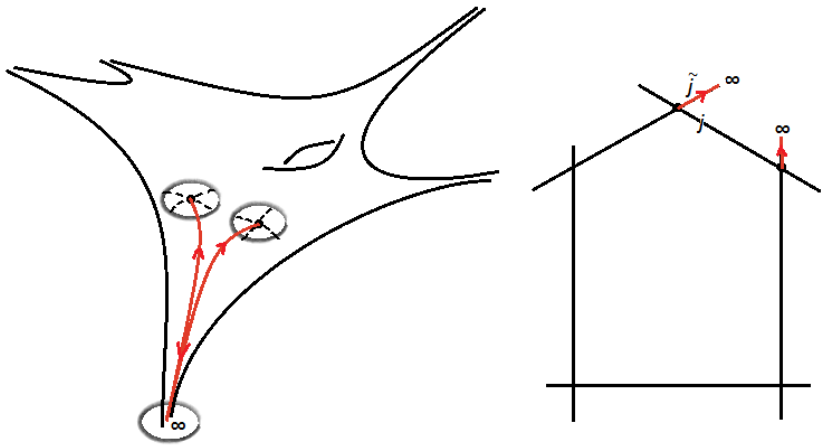

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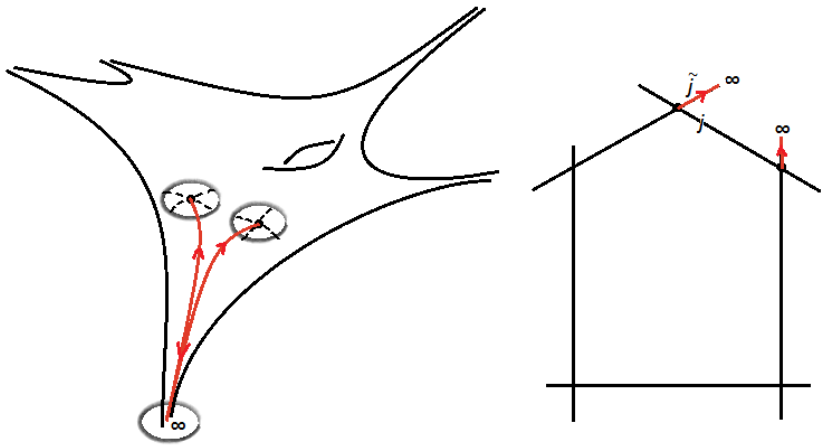

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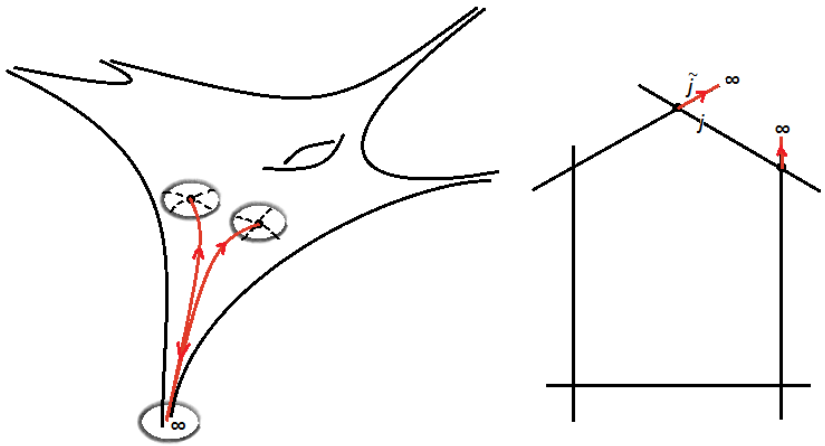

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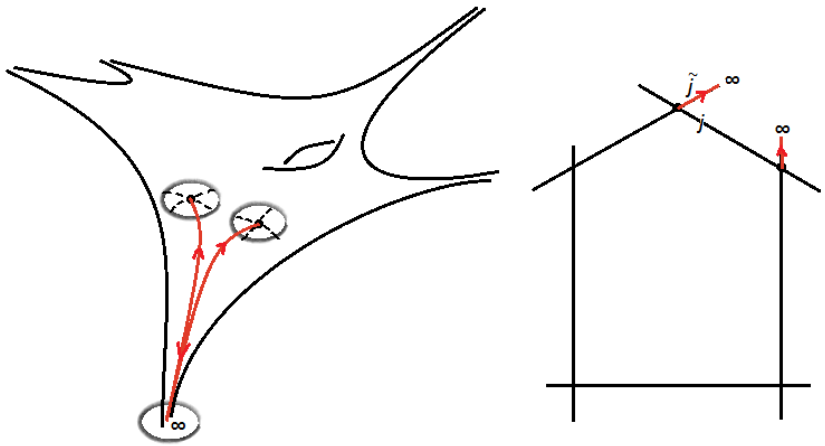

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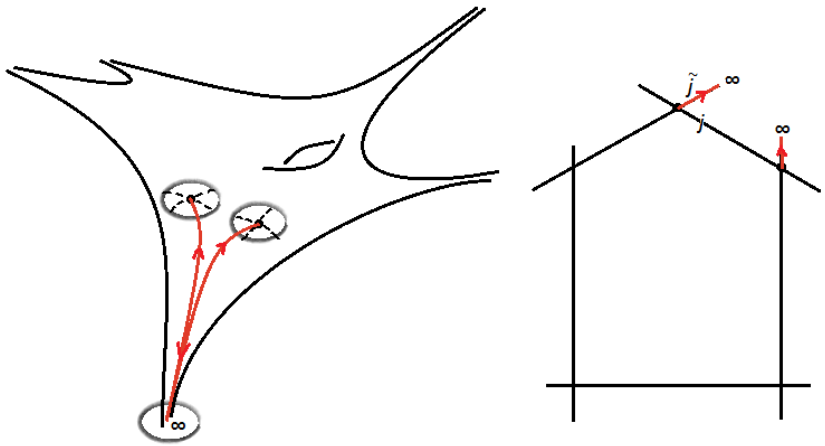

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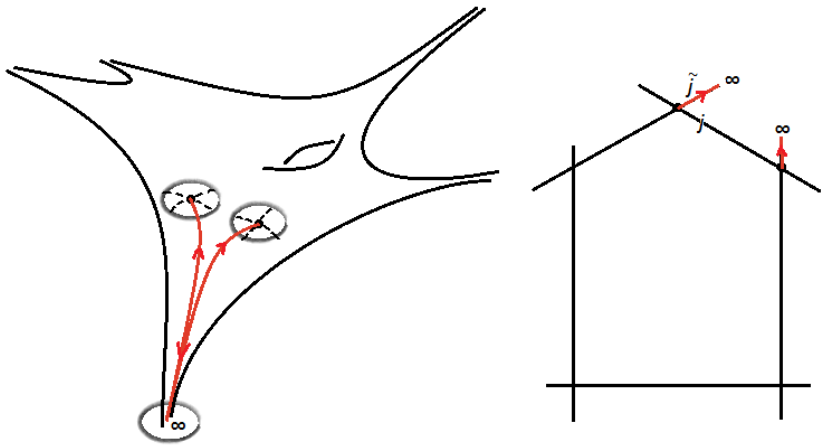

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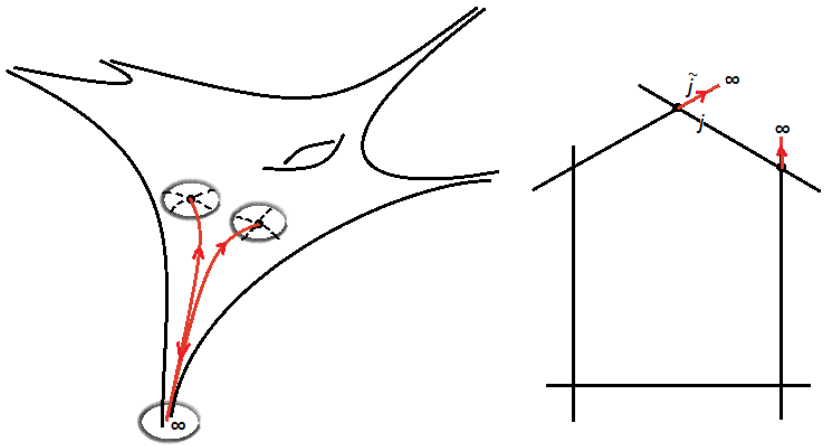

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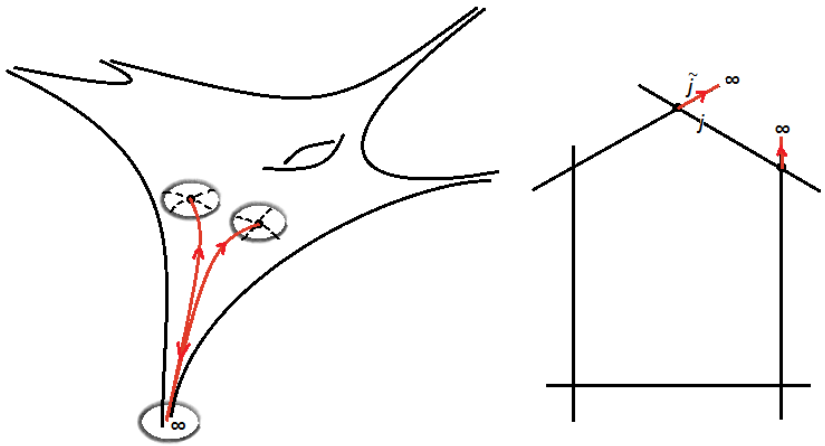

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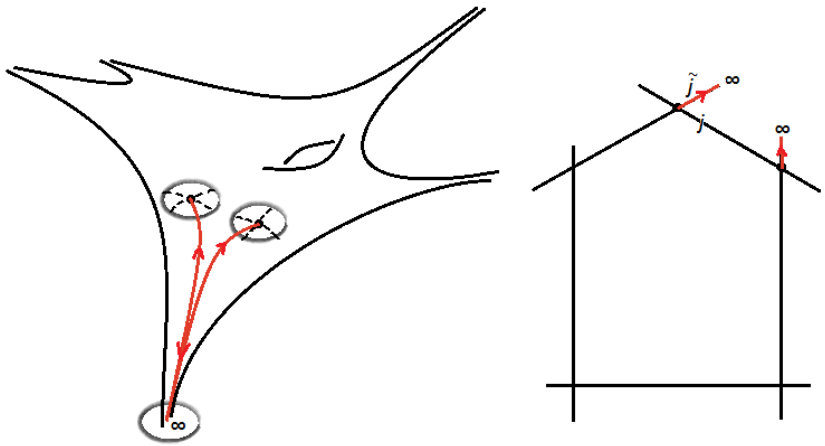

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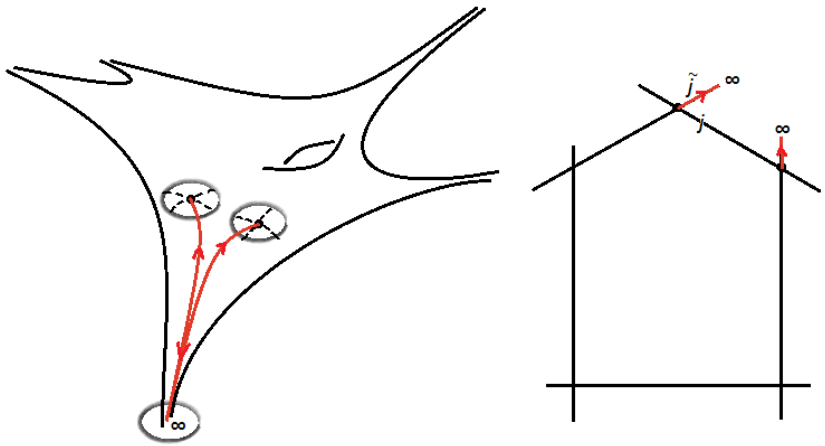

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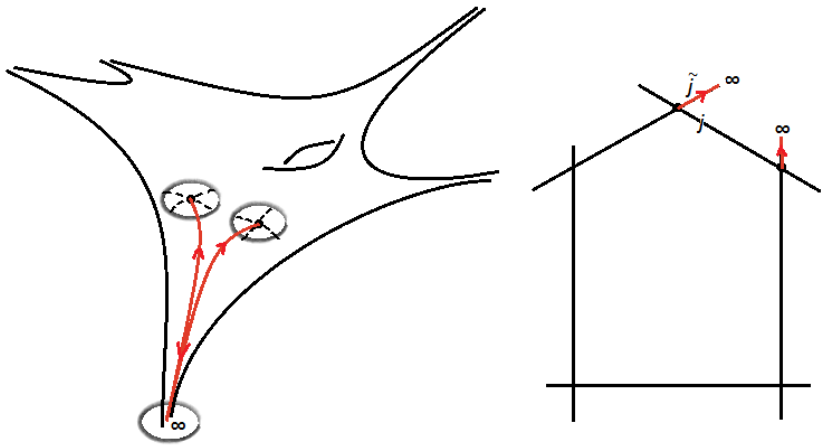

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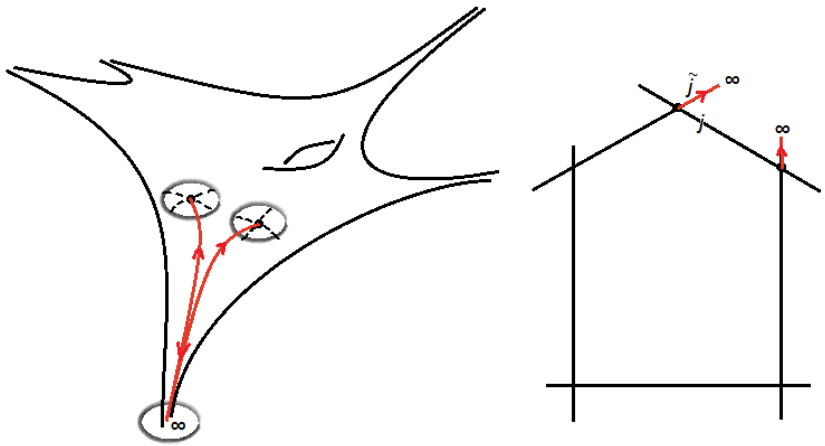

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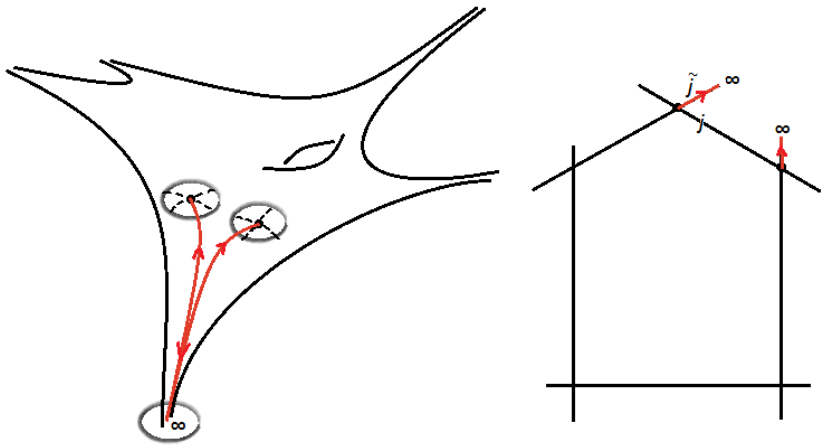

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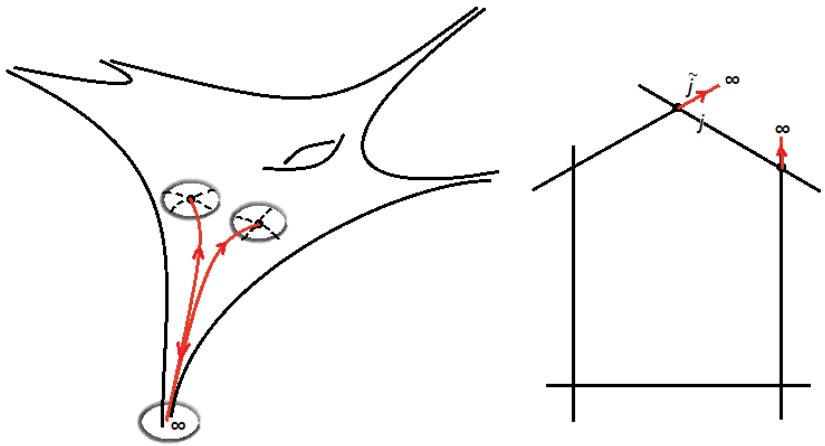

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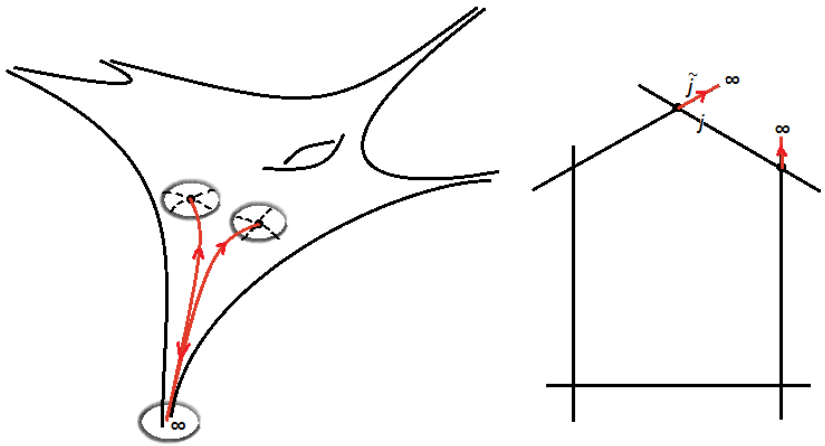

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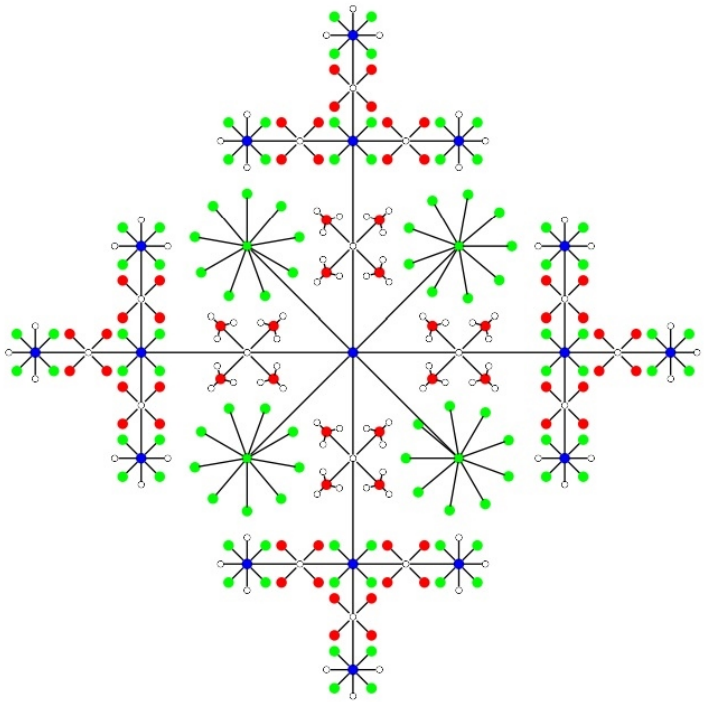
Investigating J. Weinstein's approach to integral models for modular curves via the *infinite* Lubin–Tate tower.

A picture from Jared Weinstein, Semistable models for modular curves of arbitrary level



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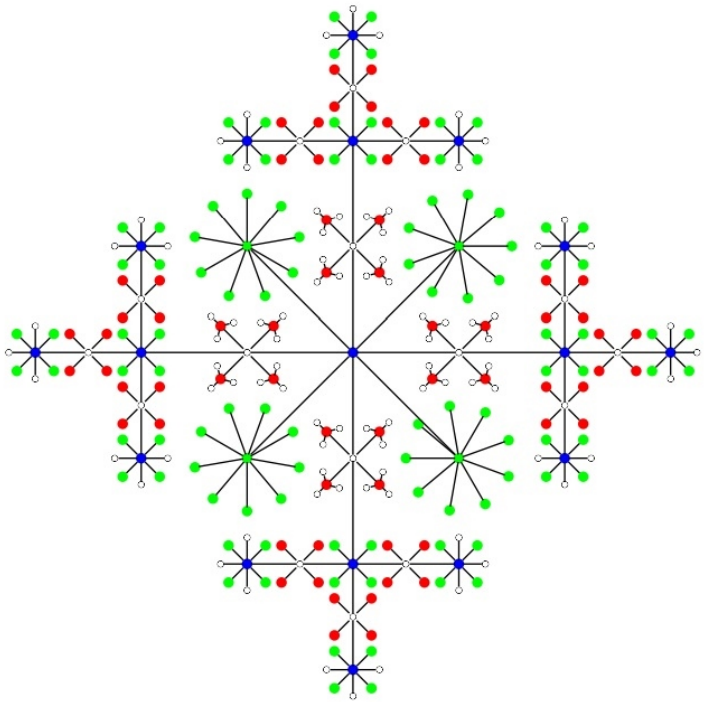

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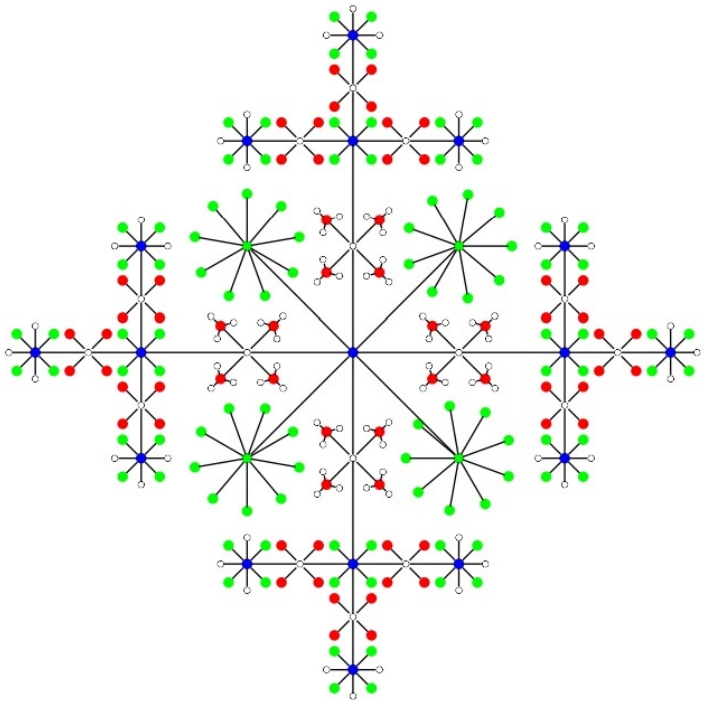

- Established in 2012, a public research-oriented university funded by the municipal government of Shenzhen, China's innovation center.
- Over 800 faculty members, 4205 undergrad students, 2214 postgrad/doctoral students, international students from 15 countries, student:teacher = 10:1.
- Set on five hundred acres of wooded landscape in the picturesque South Mountain area.
- Department of Mathematics founded in June 2015.
  28 research-and-teaching-line faculty members, 6 teaching-line faculty members, 101 grad students, 205 undergrad majors.
- International Center for Mathematics founded in February 2019.

Thank you.