International Day of Mathematics lecture, SUSTech, 2022

Weird surfaces: Möbius band, Klein bottle, and swallowtail

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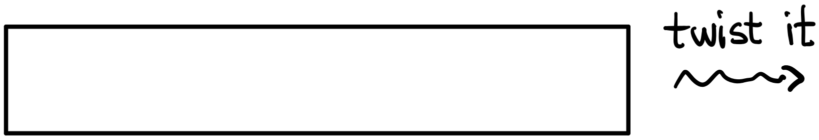

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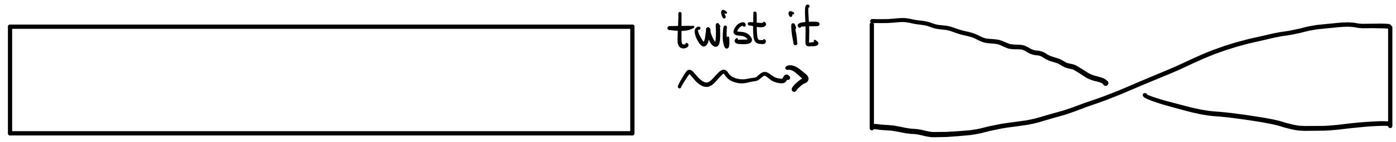

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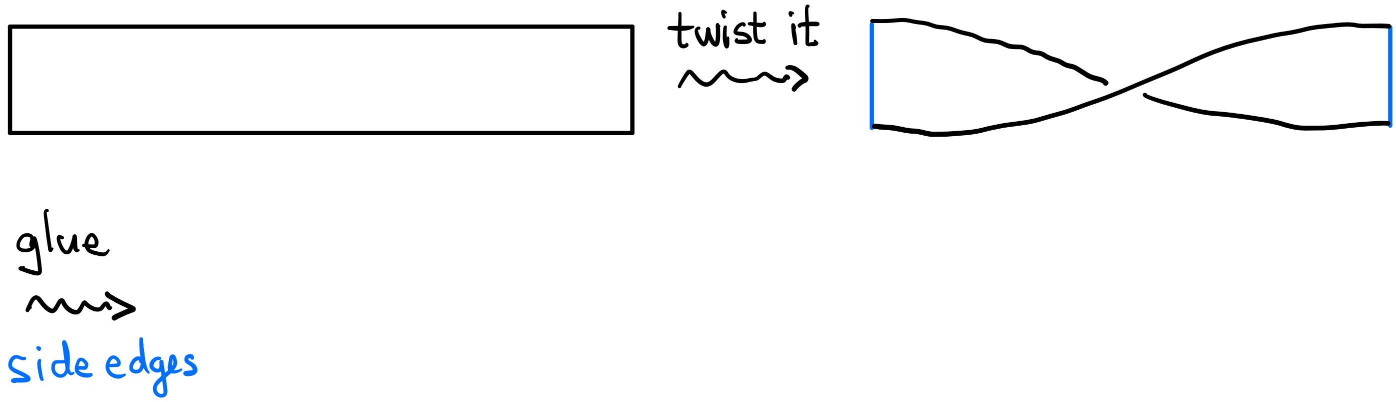

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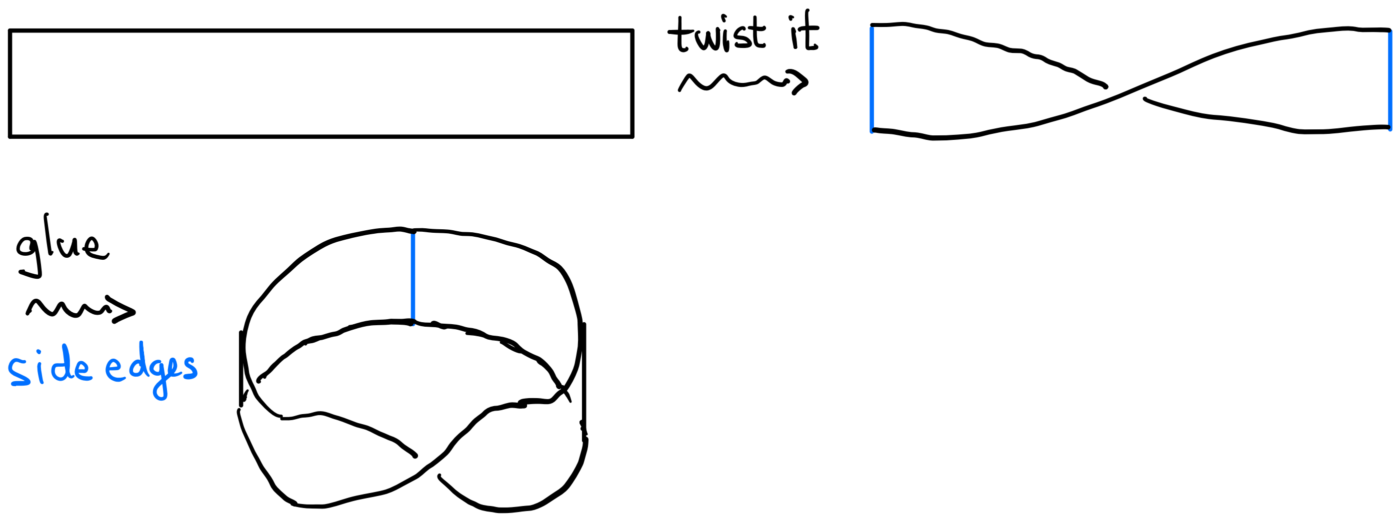

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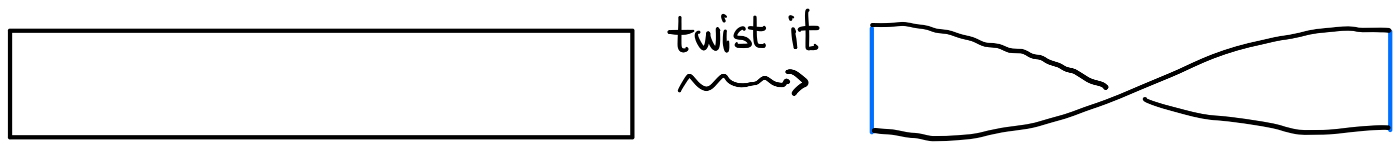

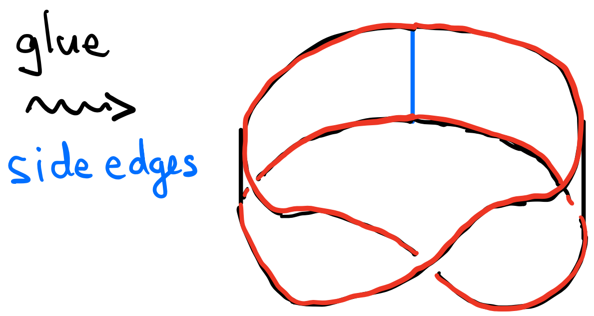

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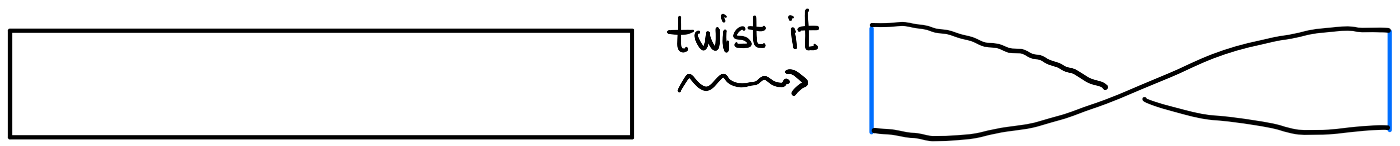

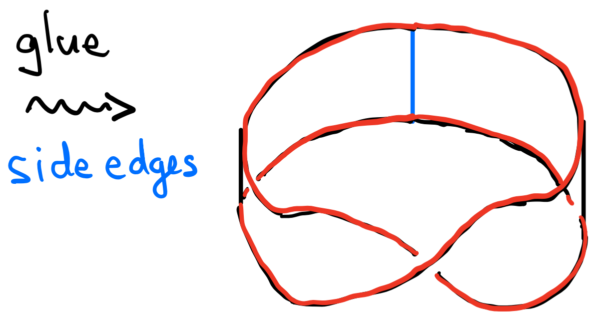

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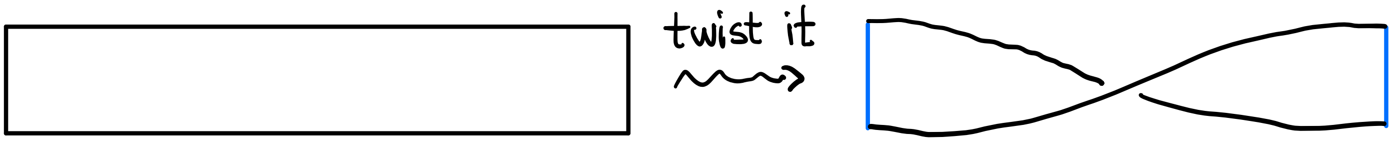

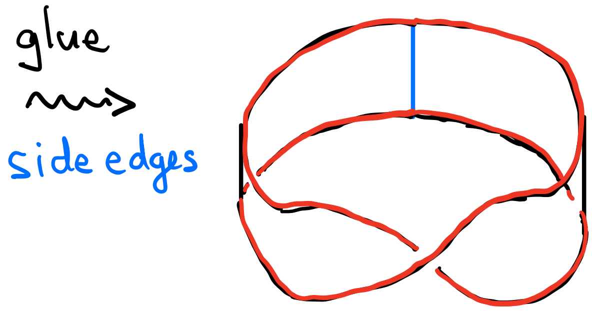

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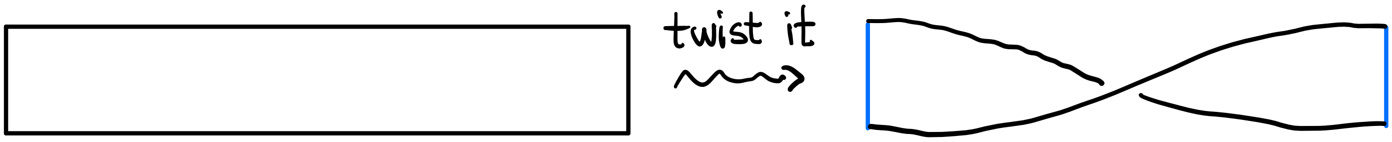

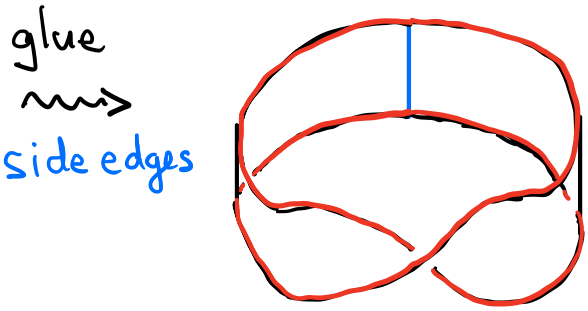

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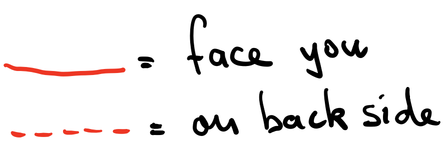

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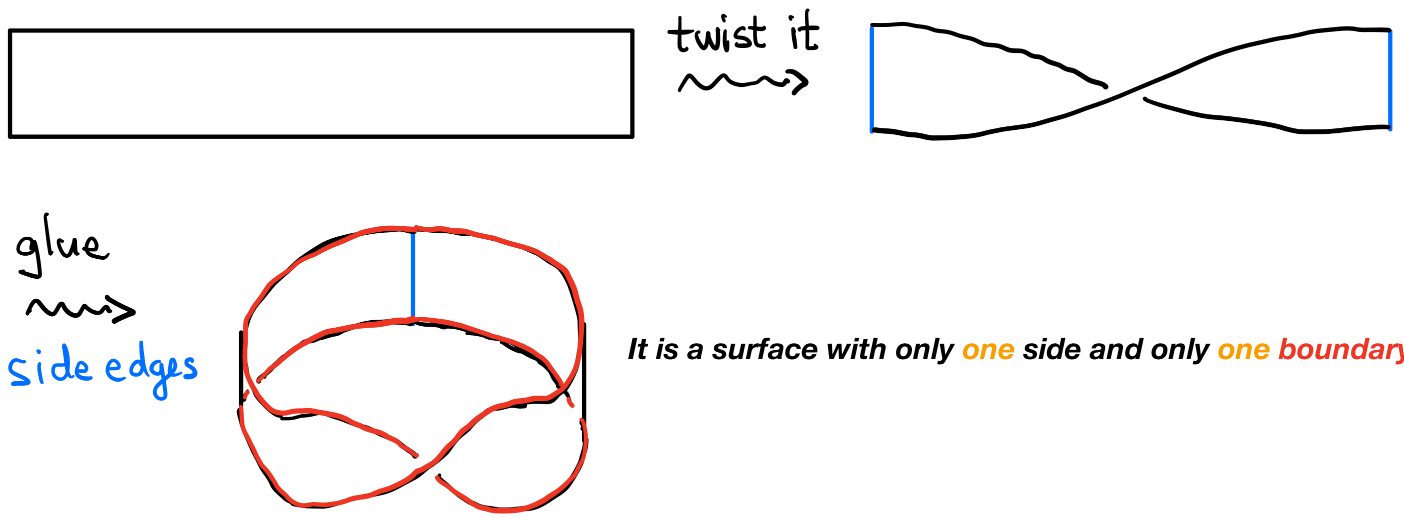

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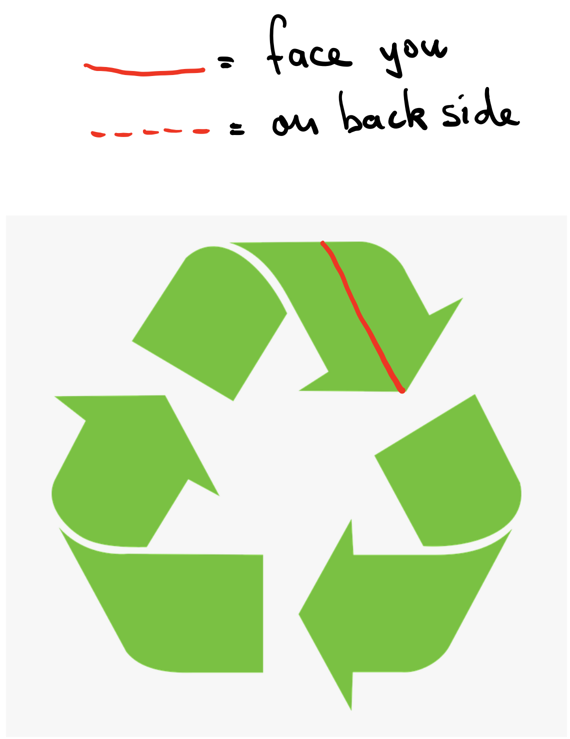

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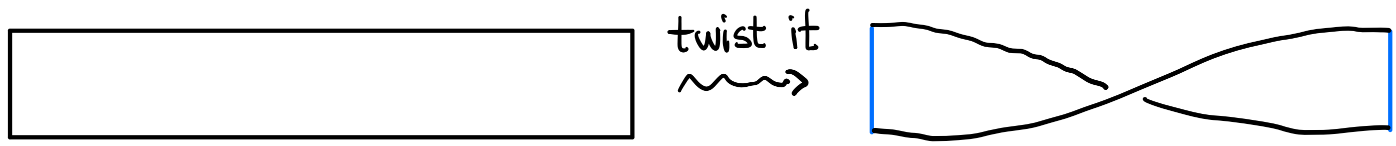

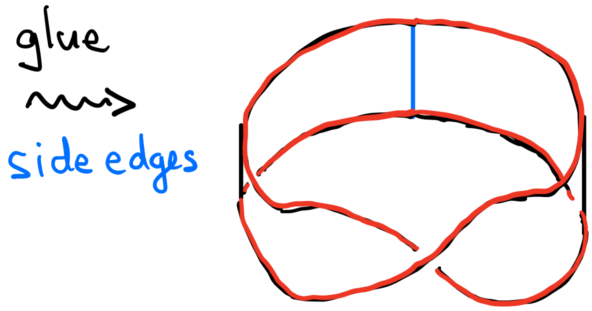

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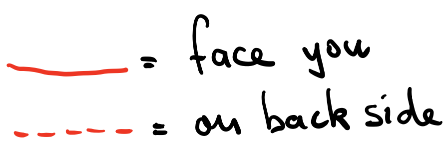

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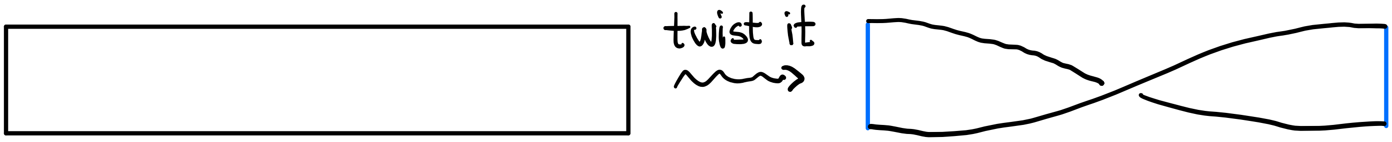

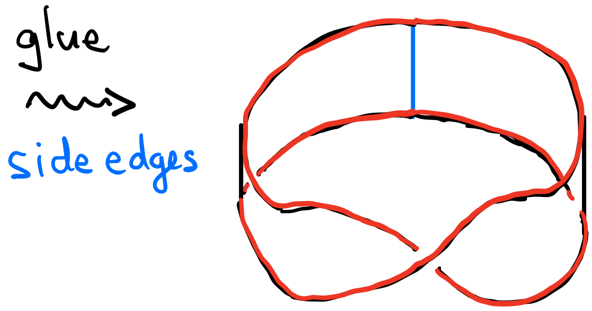

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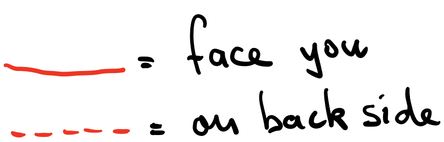

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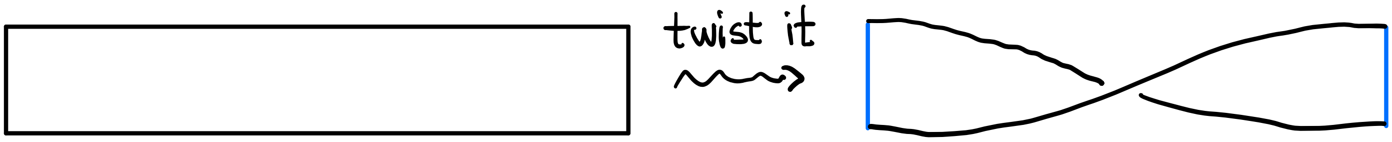

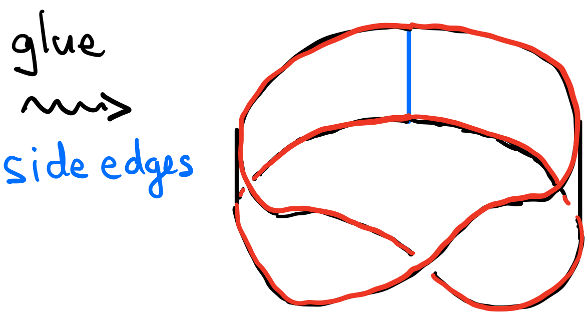

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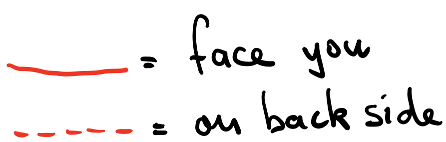

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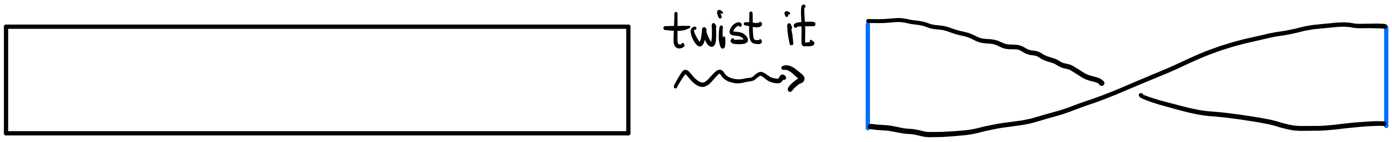

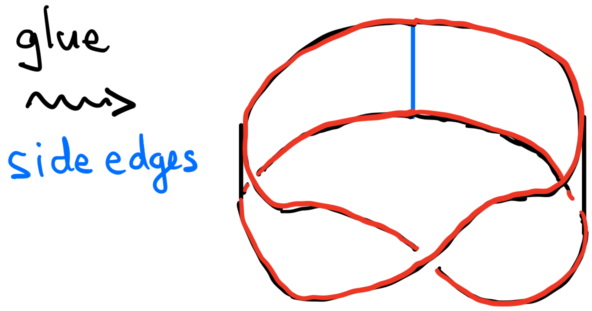

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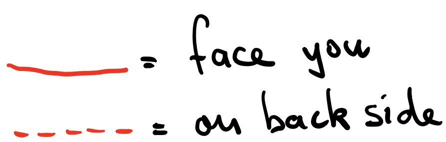

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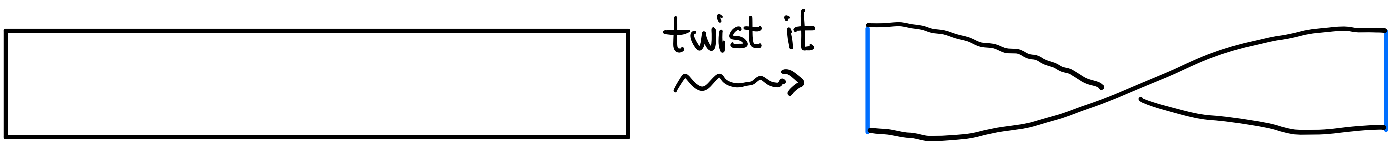

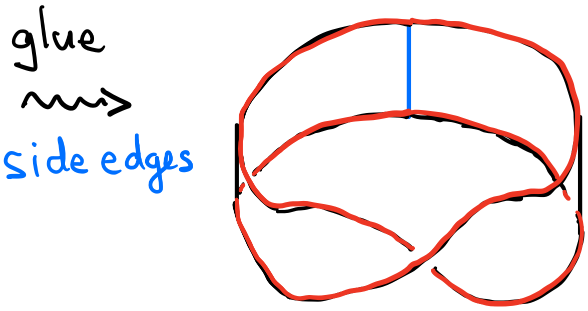

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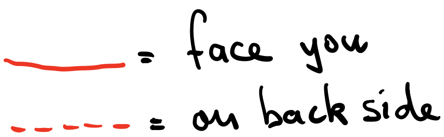

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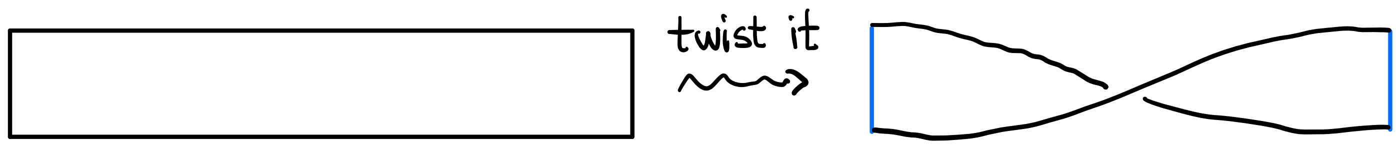

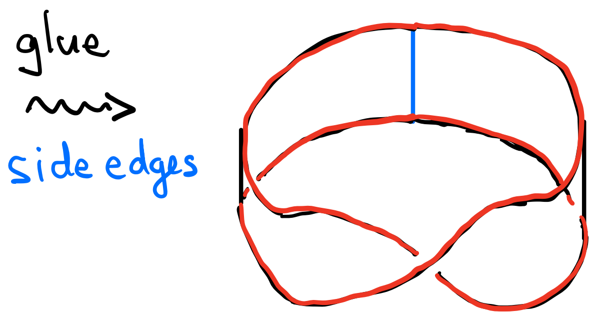

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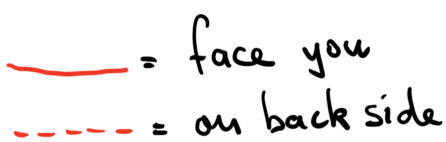

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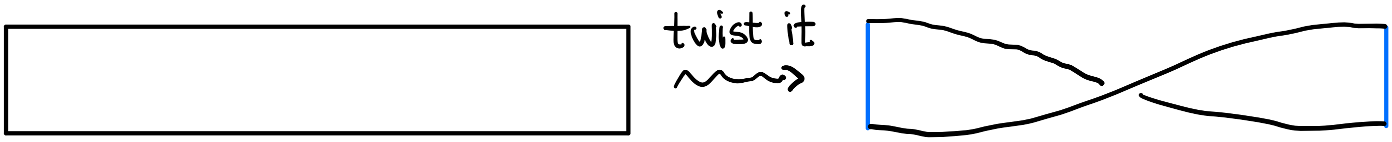

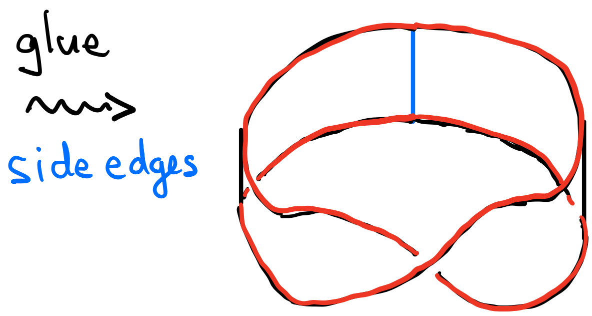

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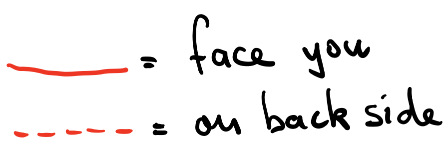

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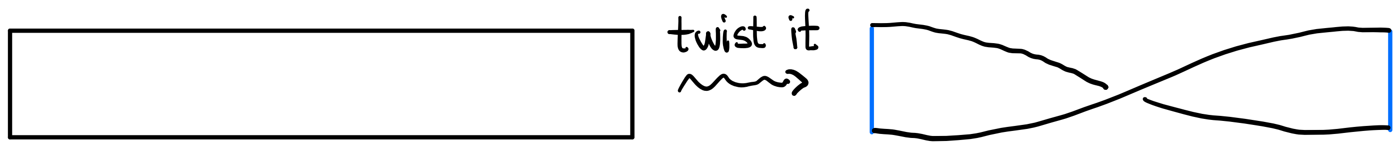

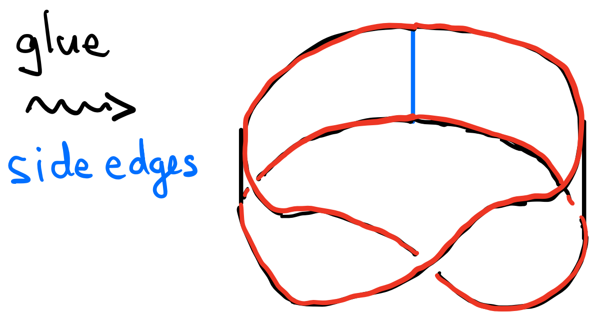

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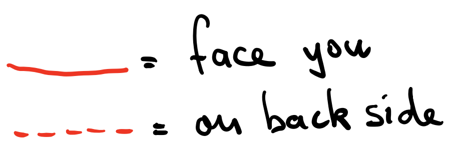

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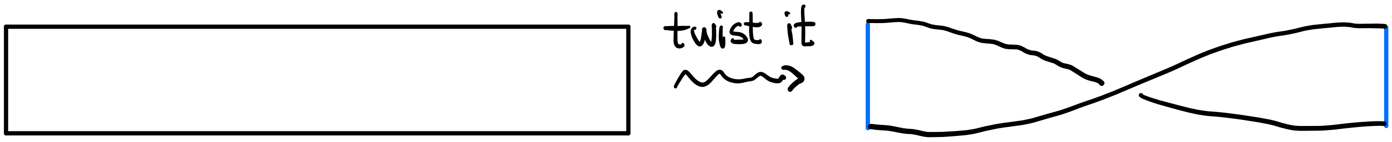

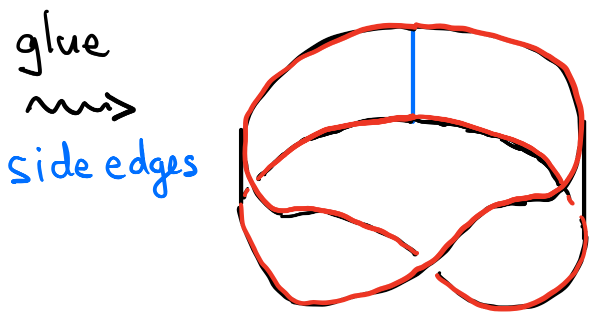

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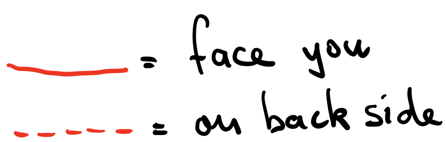

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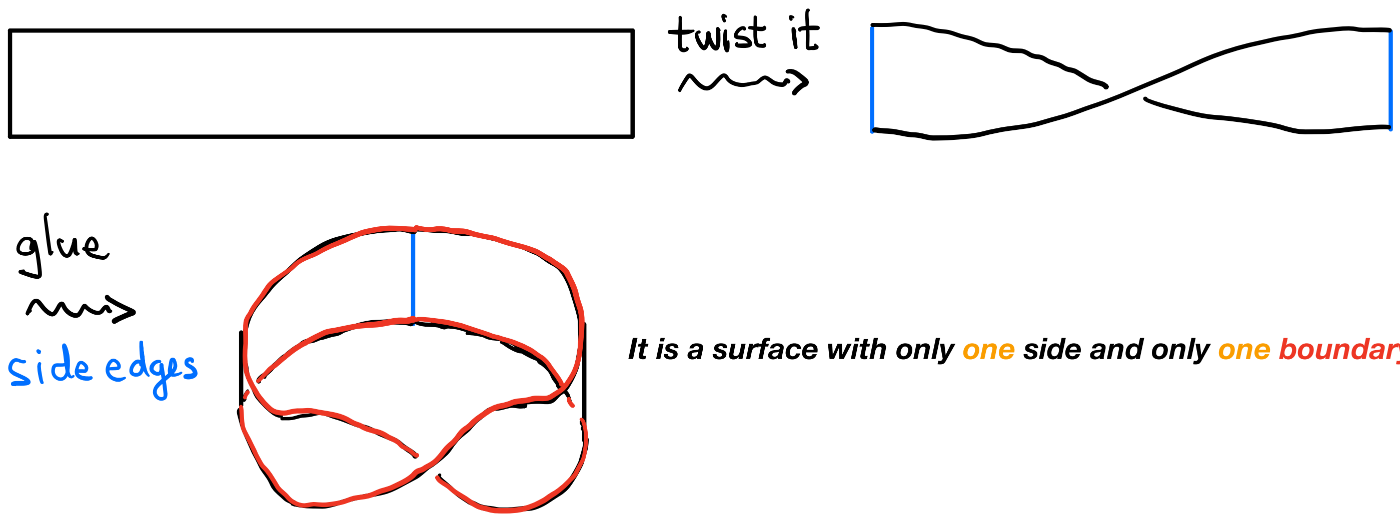

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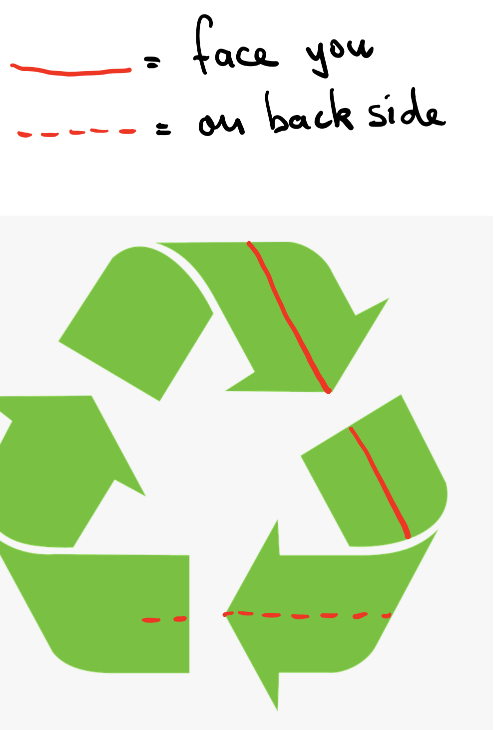

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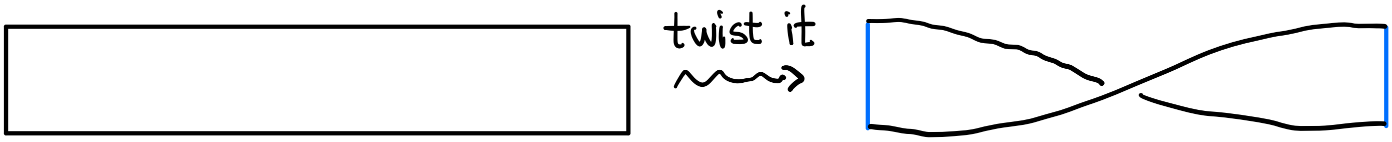

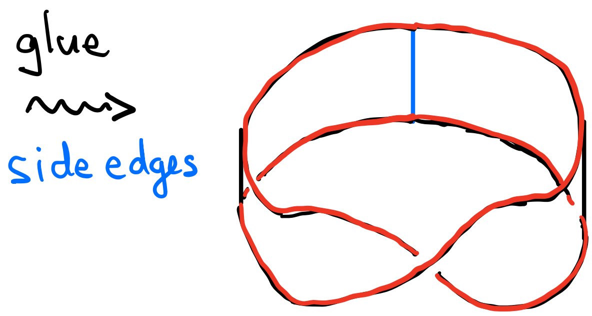

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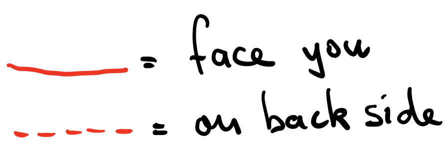

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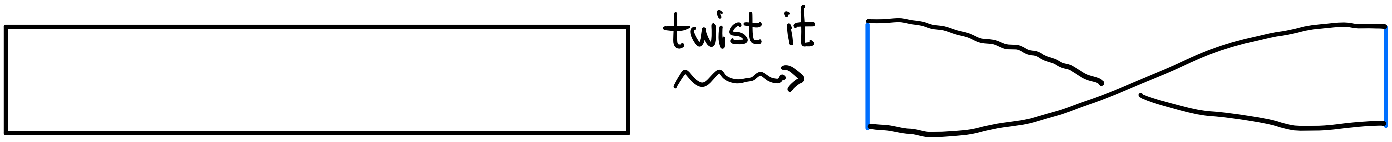

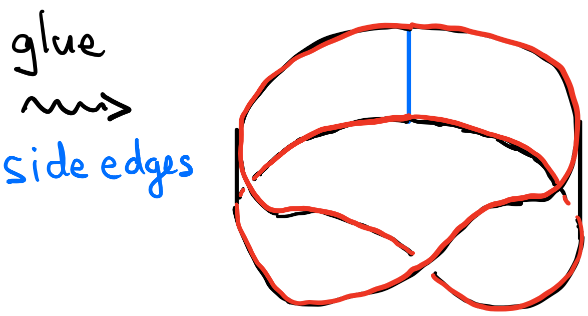

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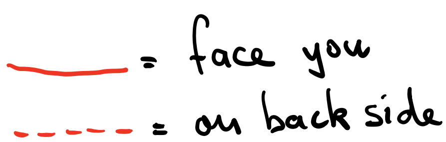

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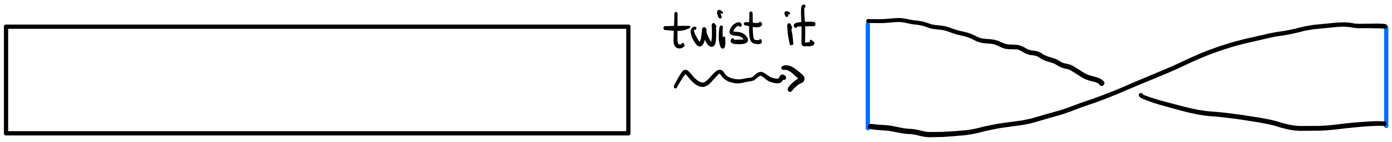

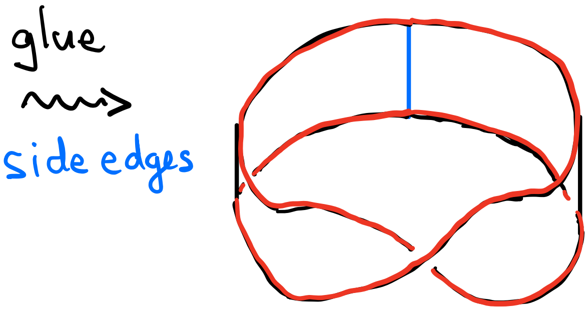

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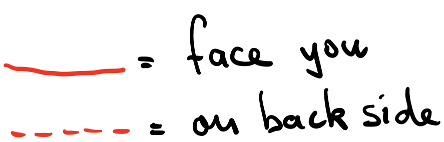

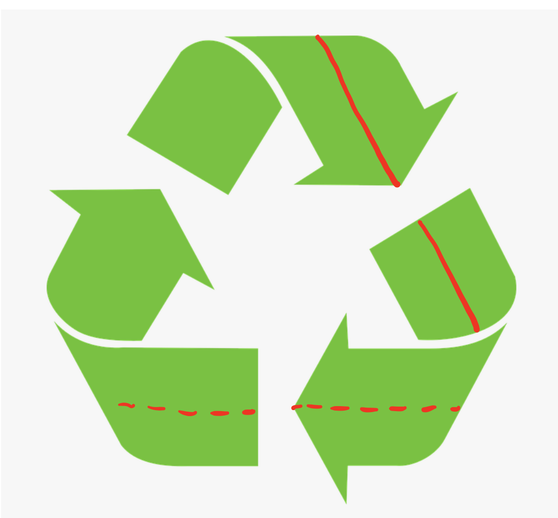

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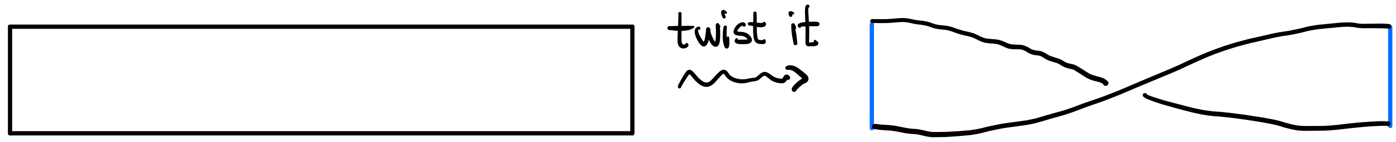

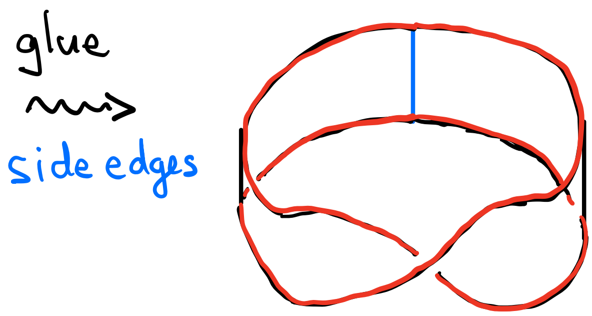

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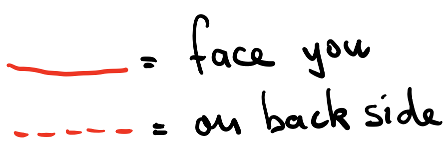

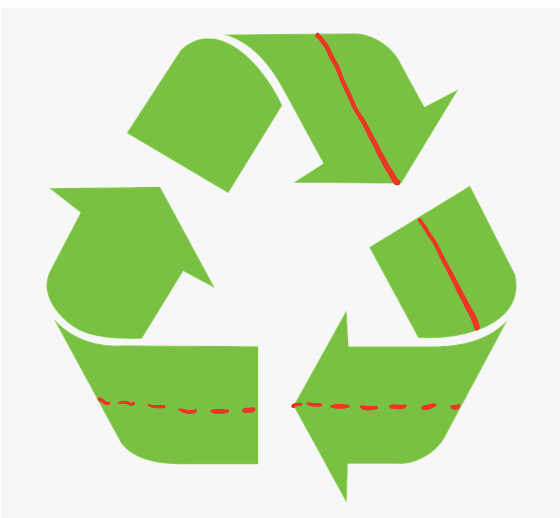

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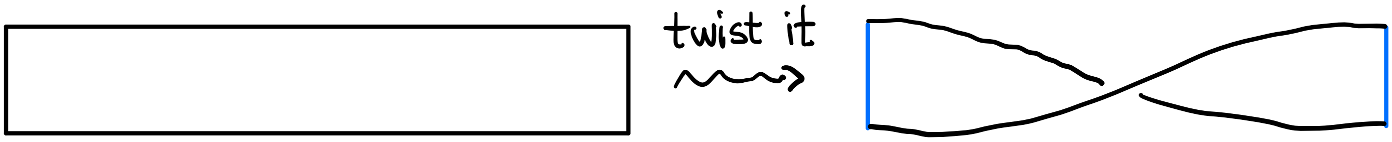

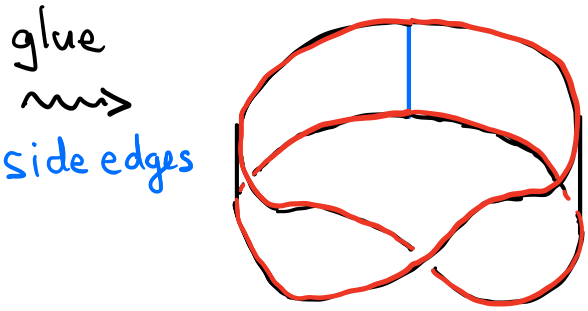

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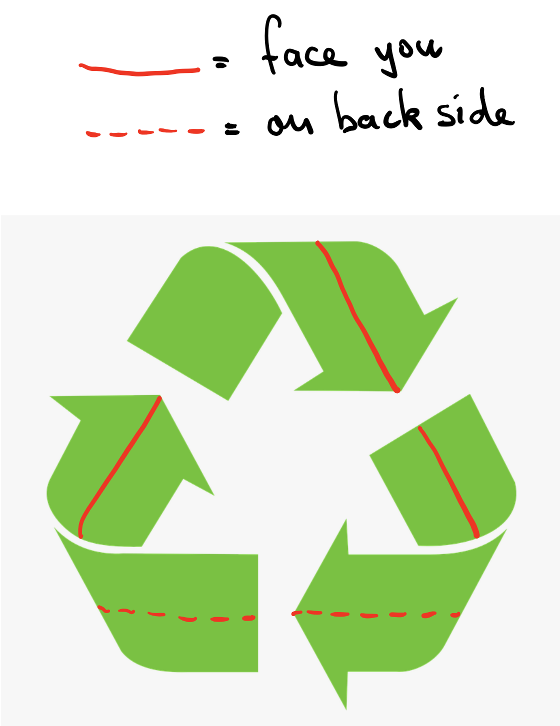

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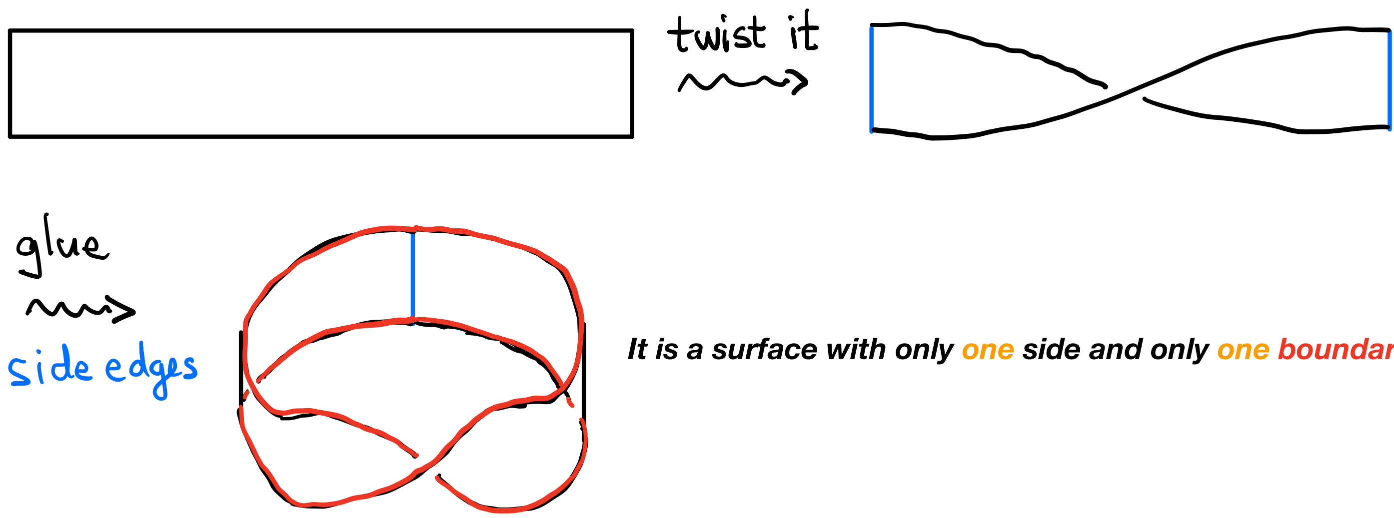

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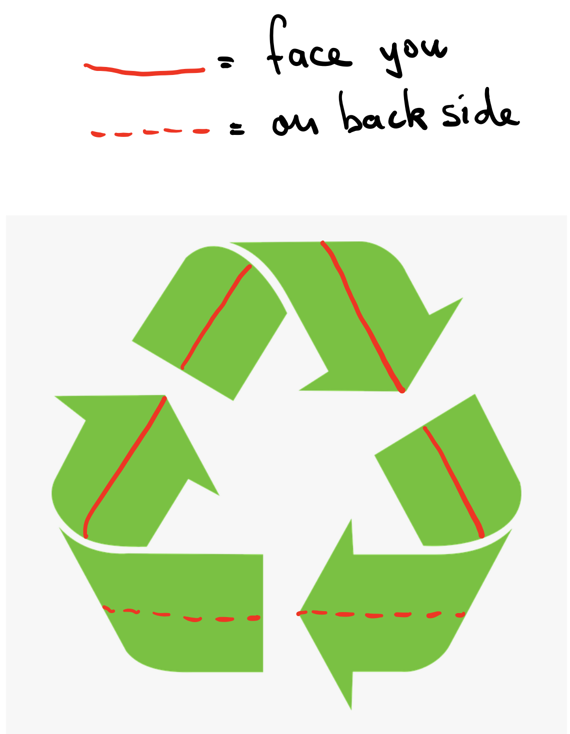

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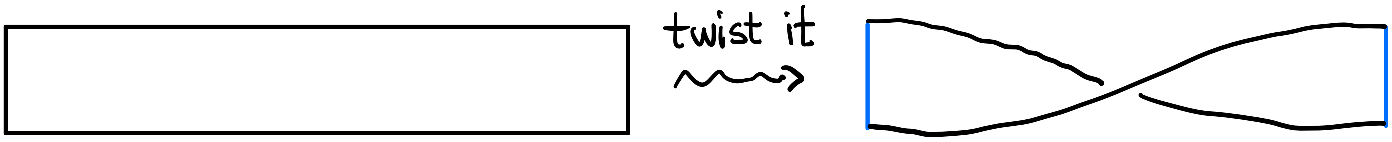

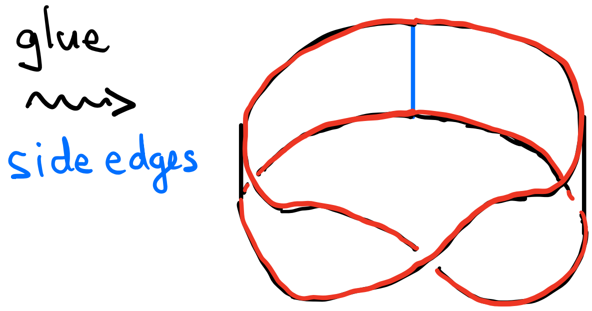

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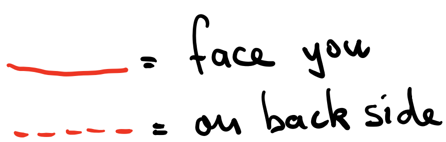

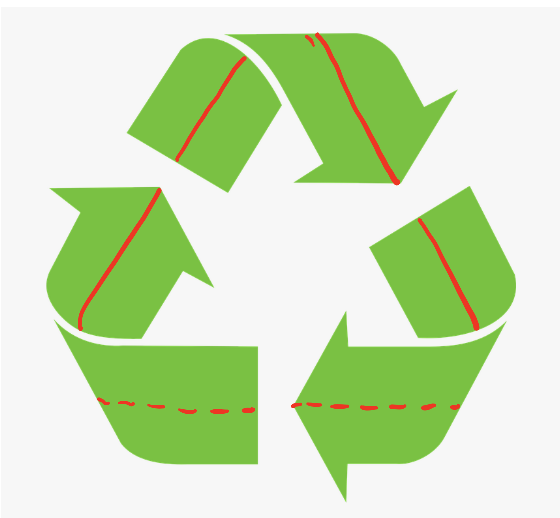

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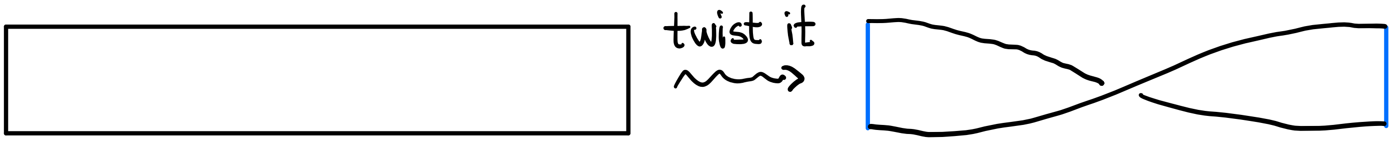

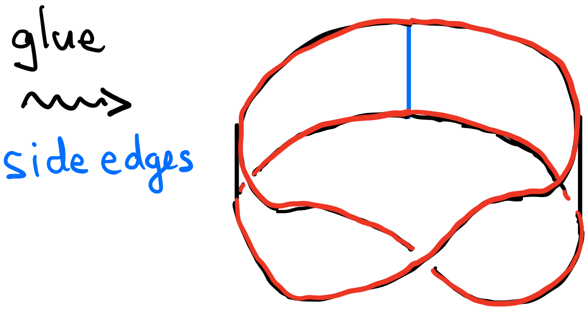

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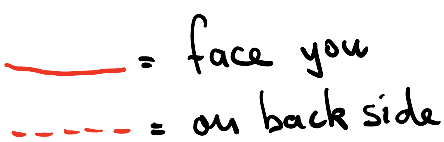

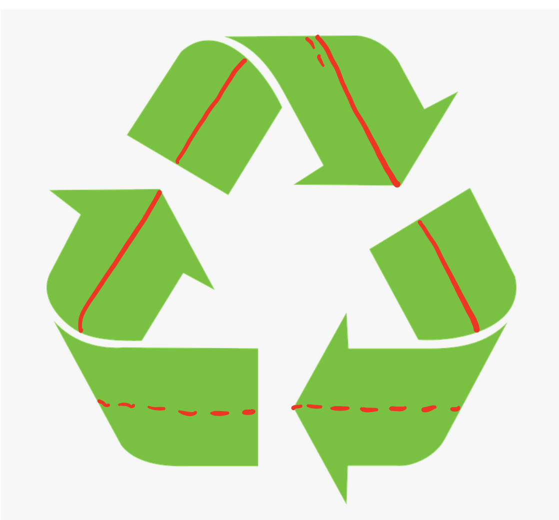

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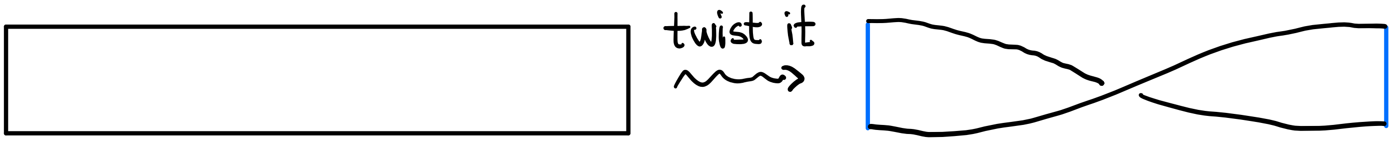

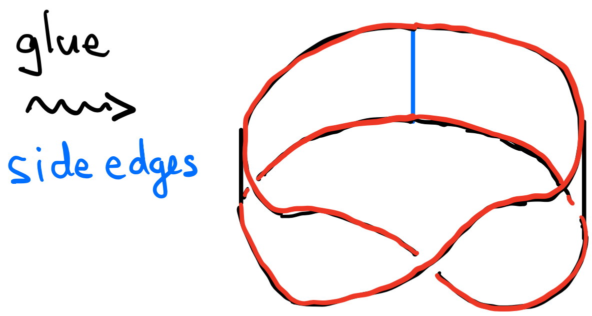

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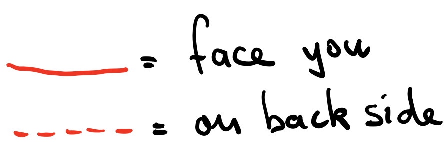

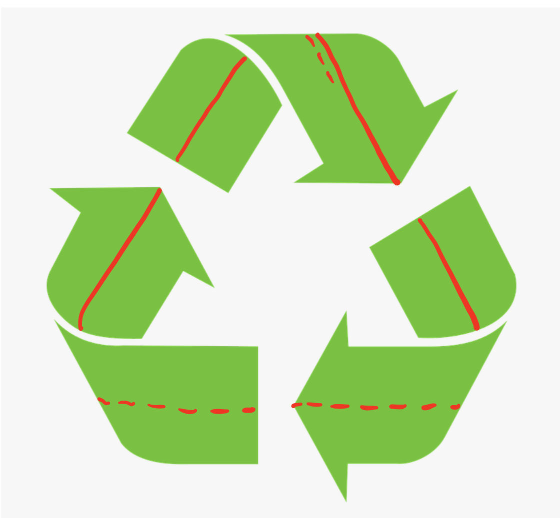

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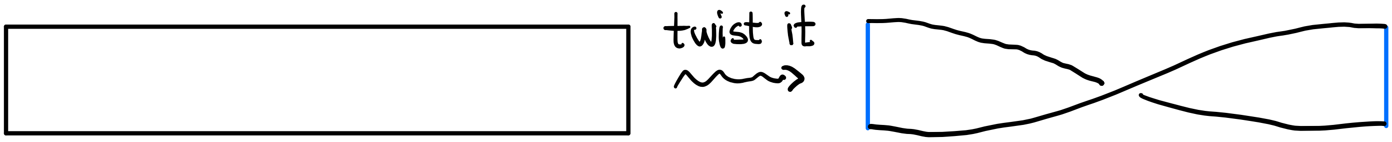

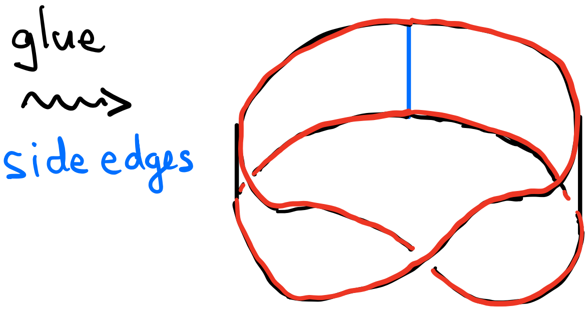

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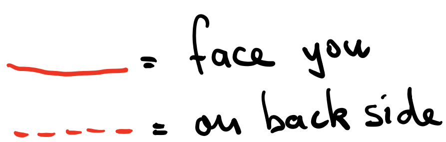

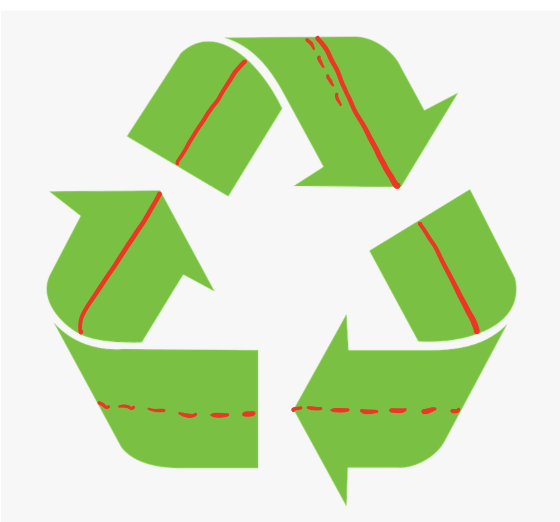

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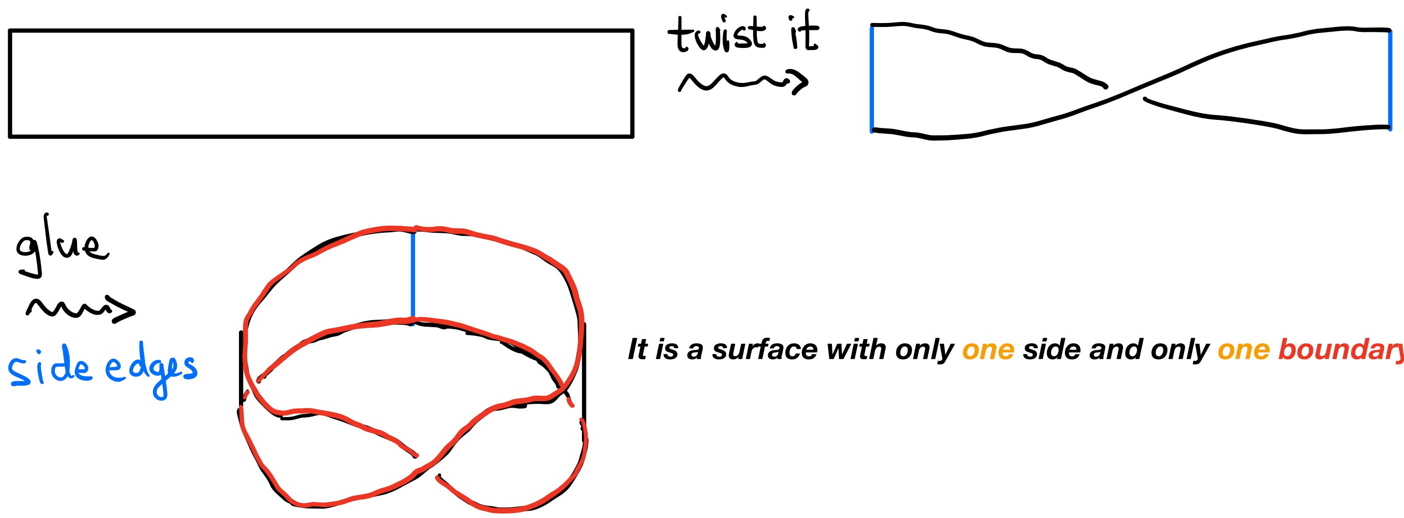

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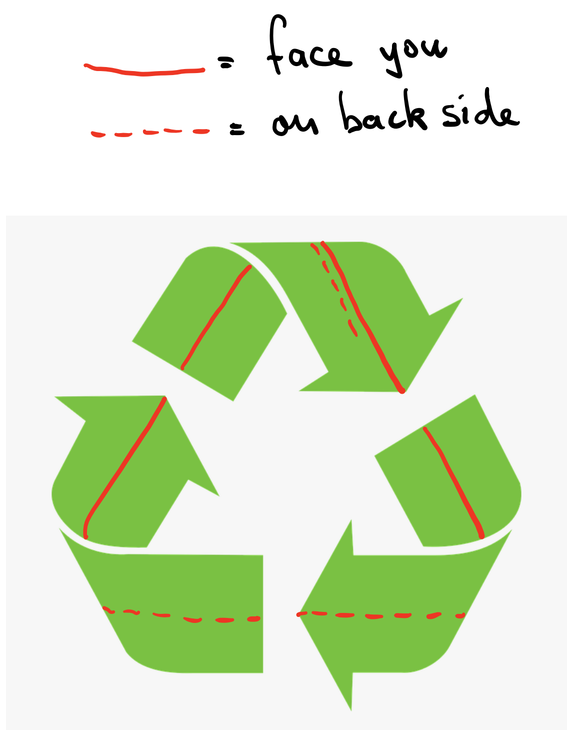

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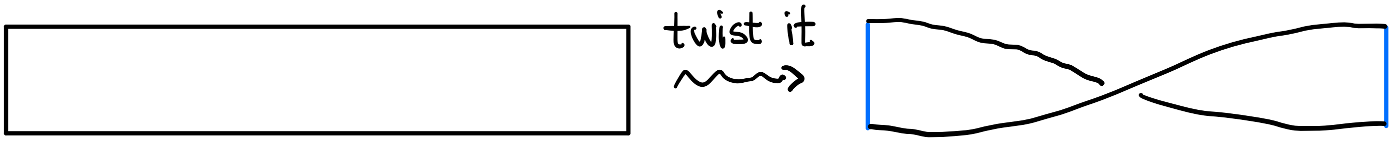

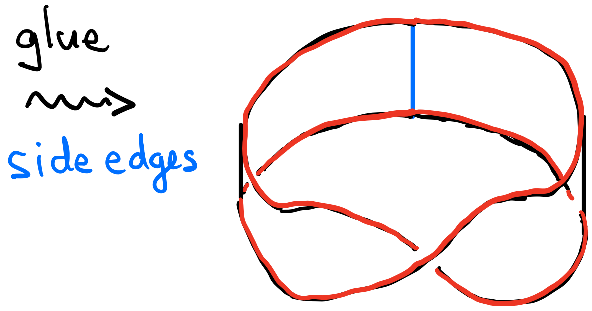

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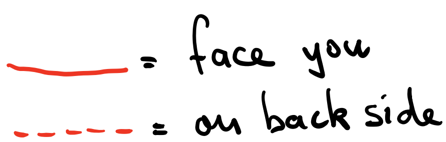

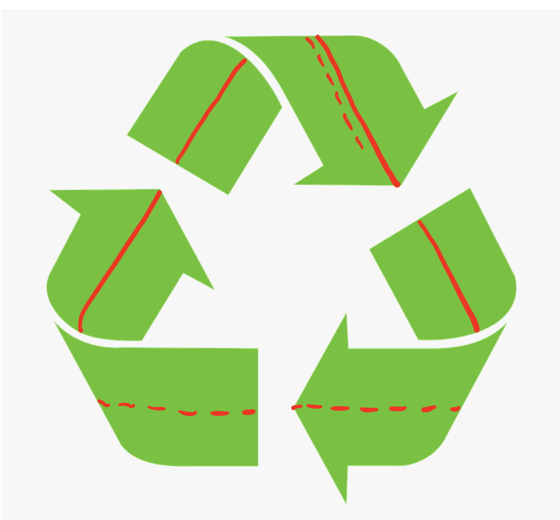

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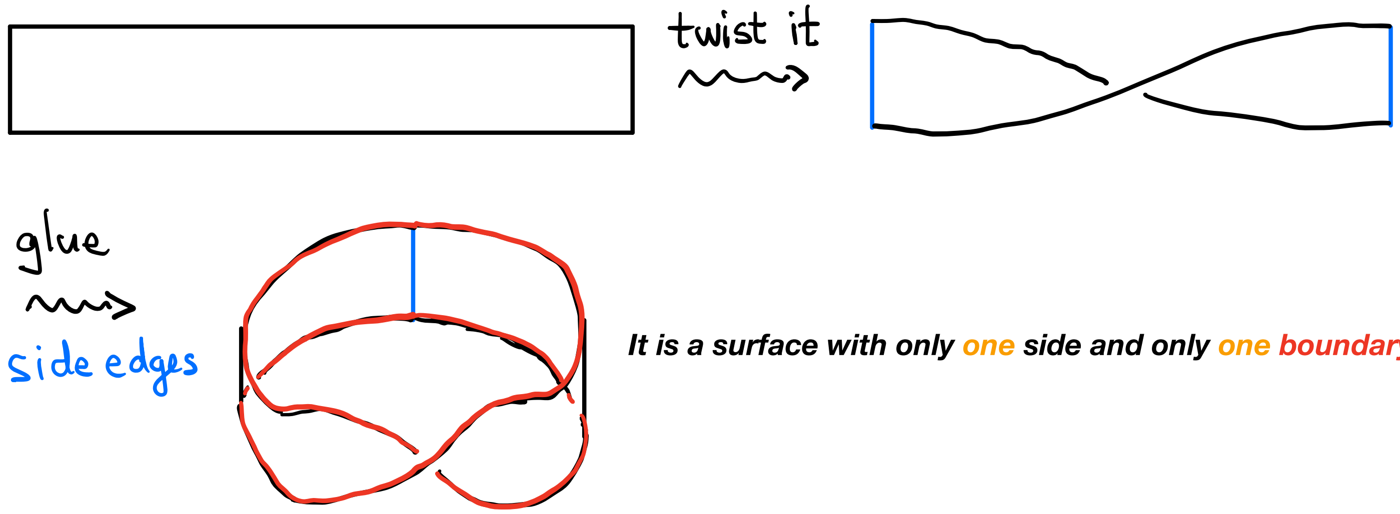

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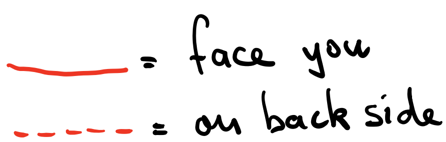

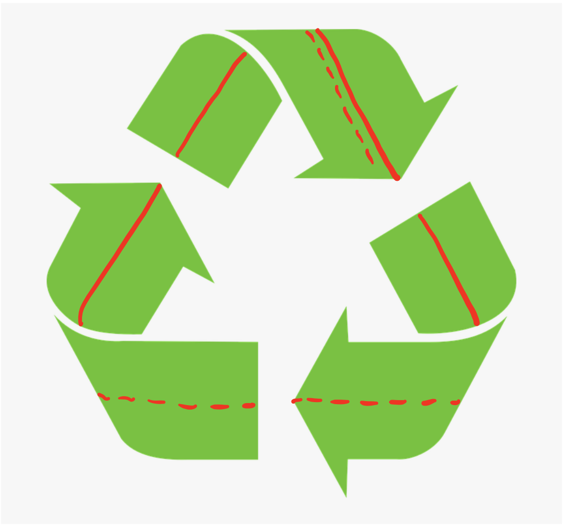

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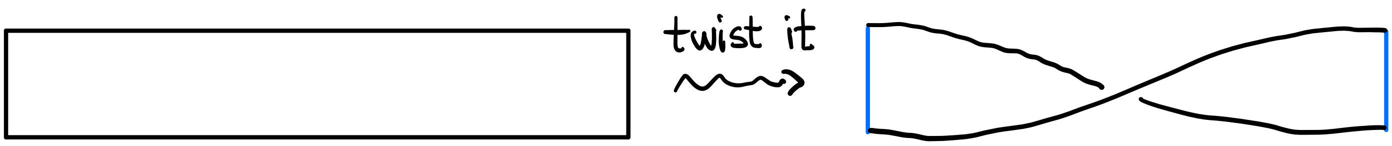

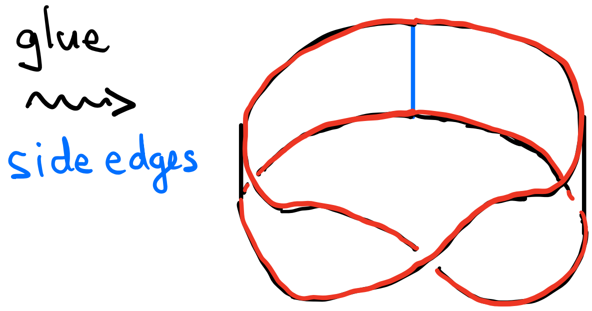

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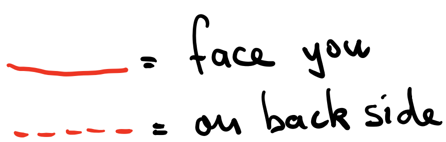

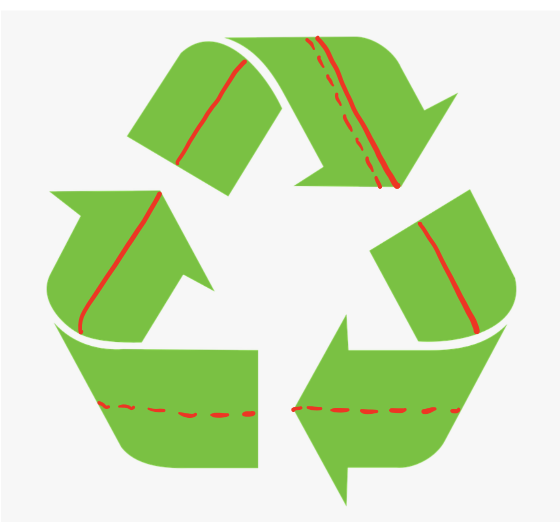

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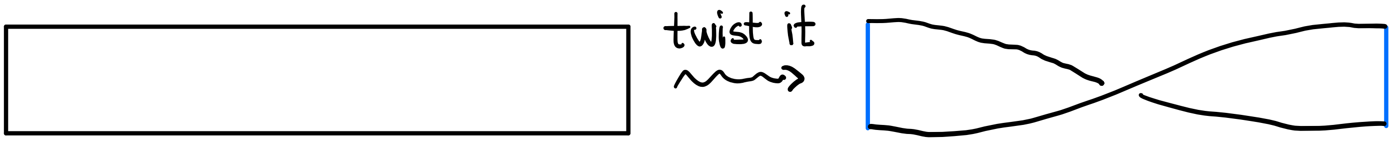

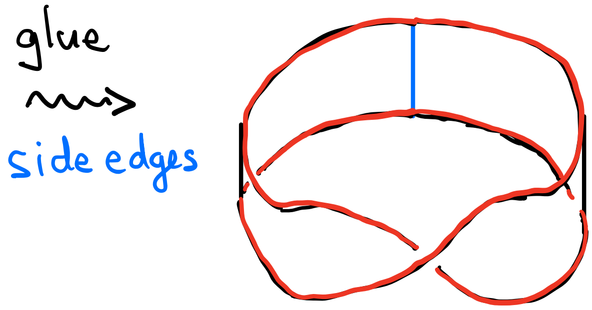

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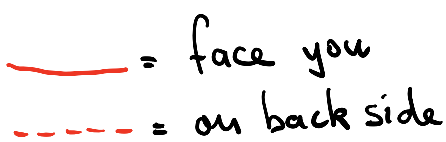

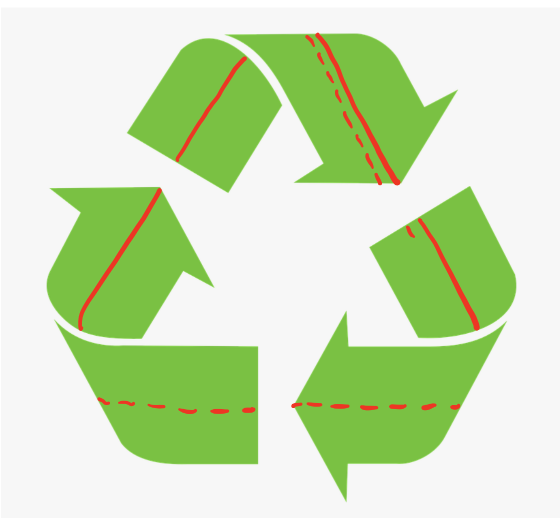

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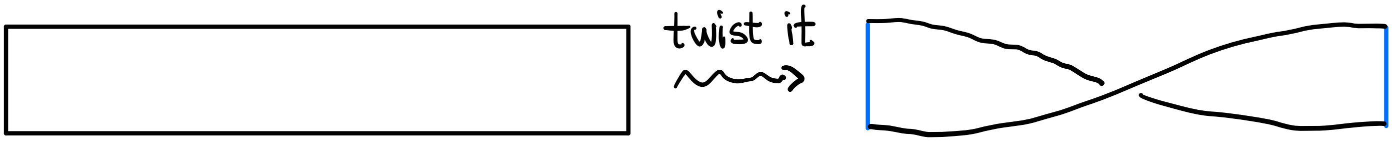

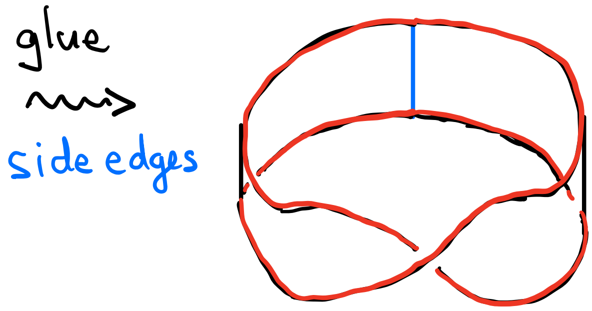

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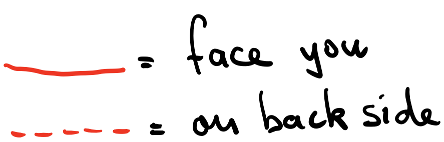

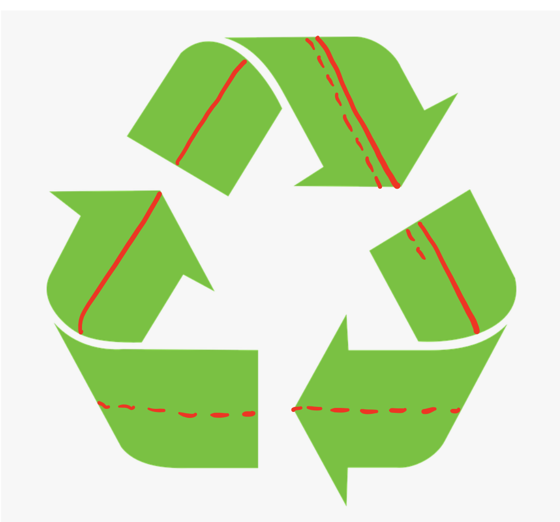

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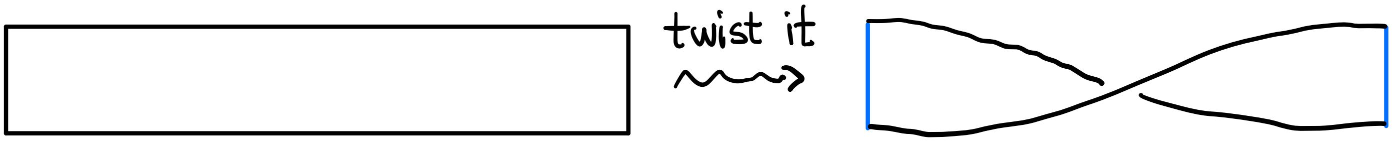

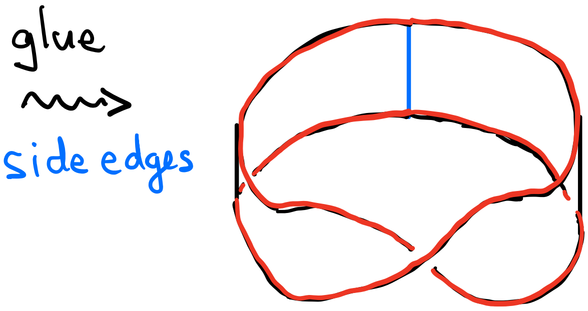

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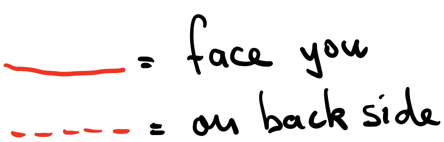

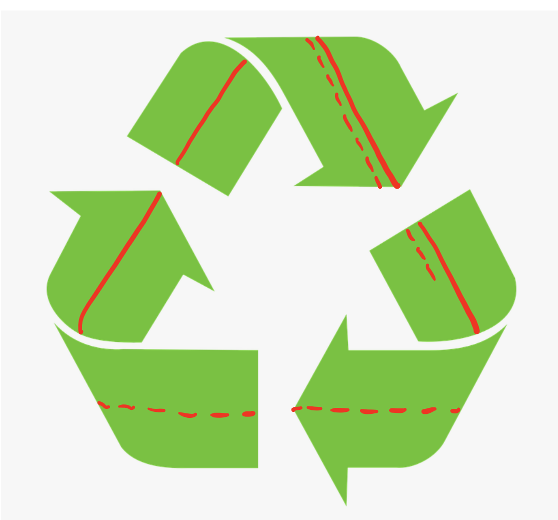

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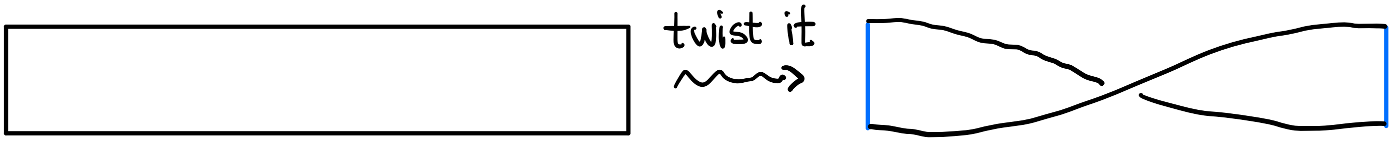

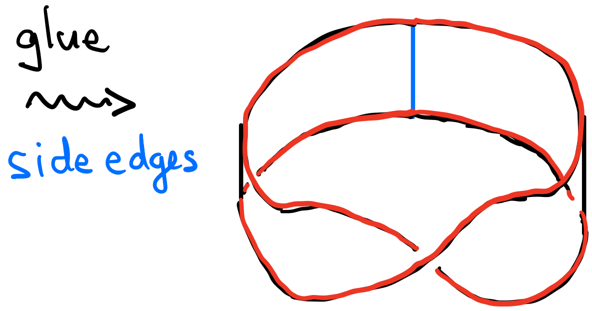

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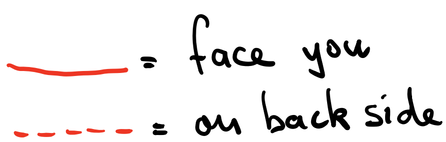

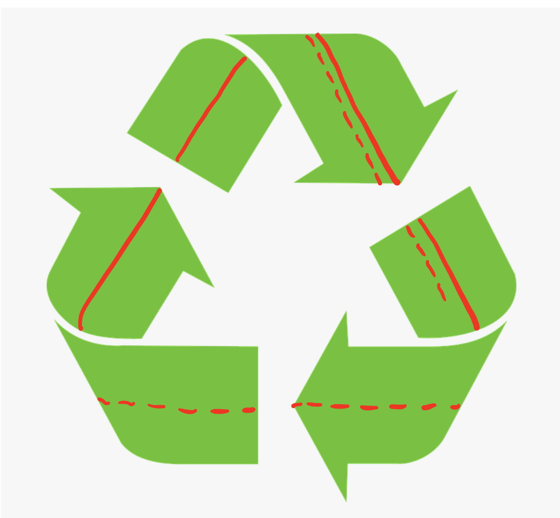

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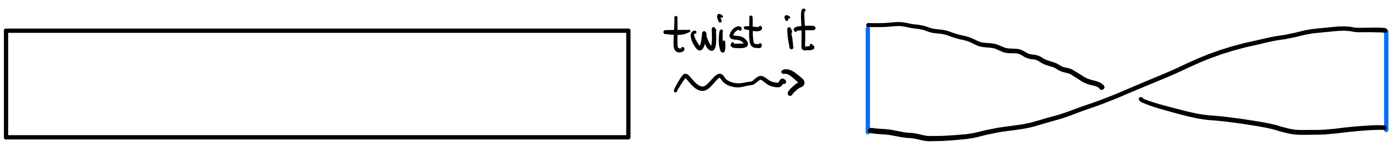

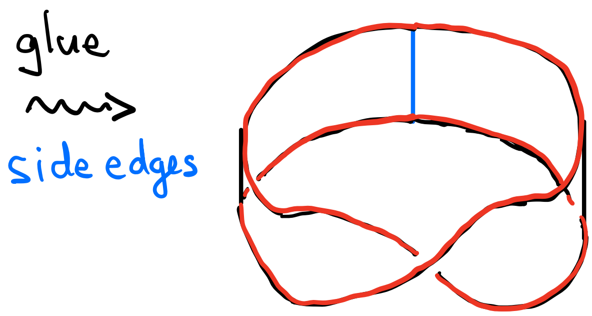

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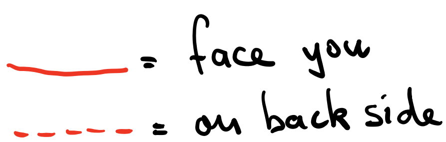
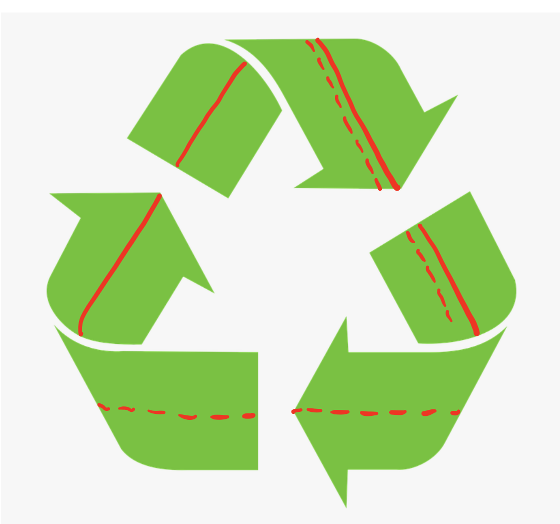
Where do we SEE a Möbius band?

Let's RECYCLE!





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Weird surfaces: Möbius band, Klein bottle, and swallowtail

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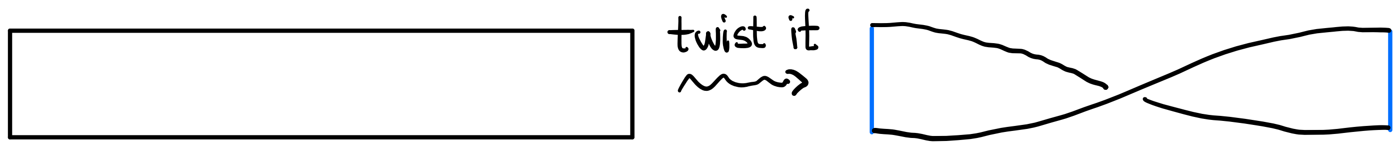

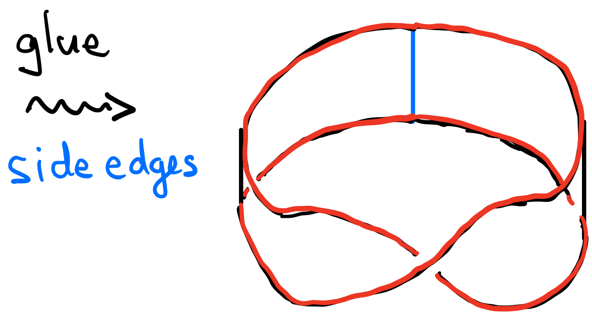

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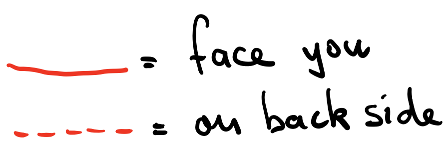

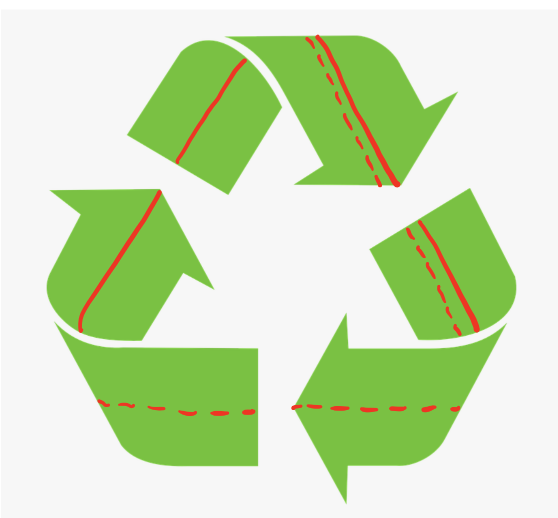

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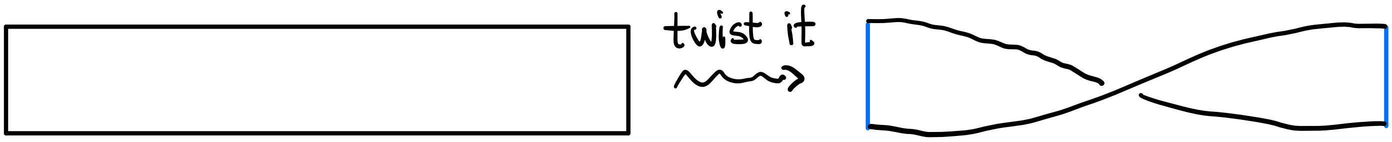

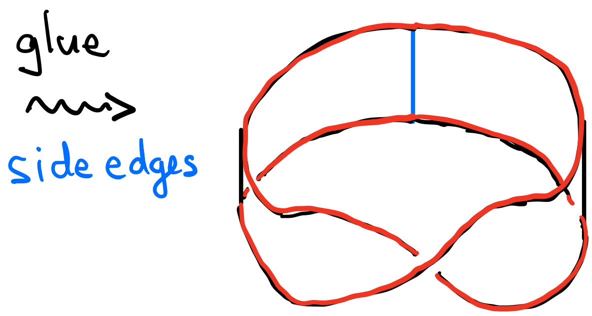

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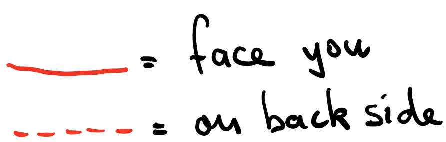

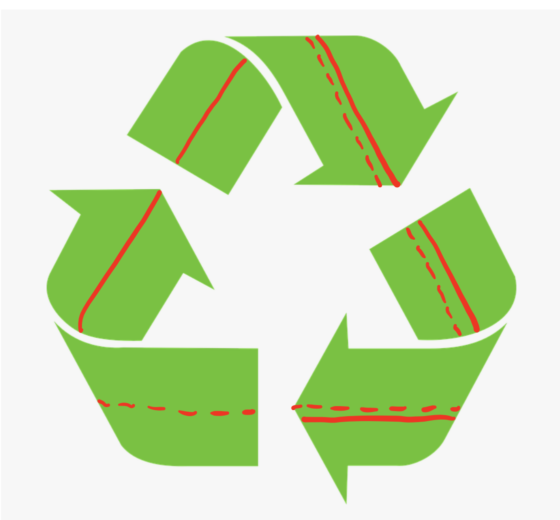

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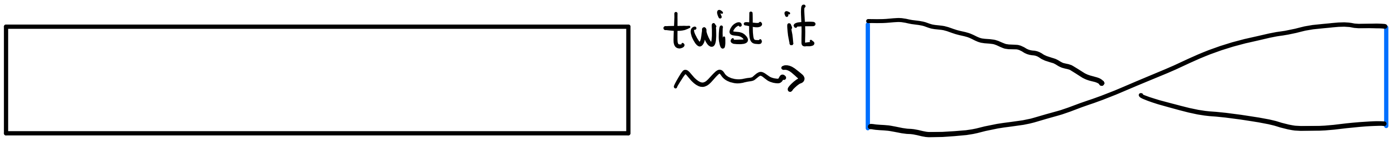

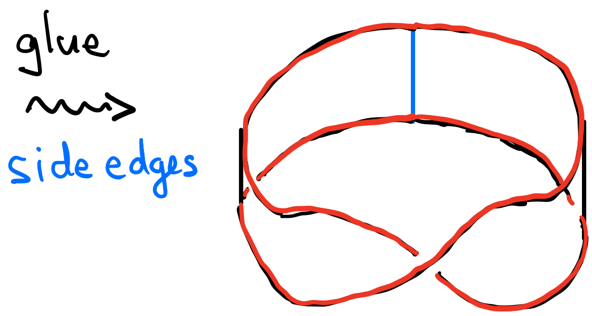

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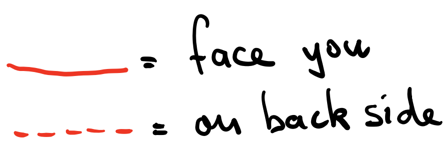

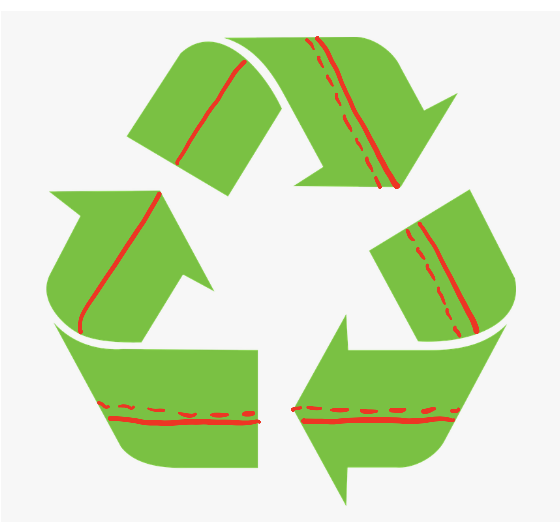

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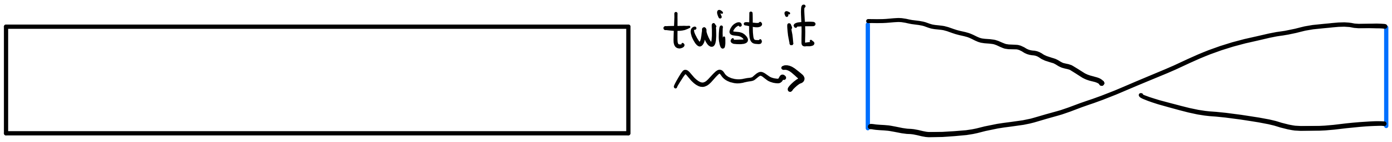

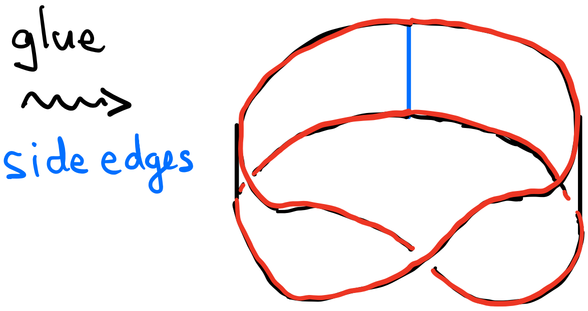

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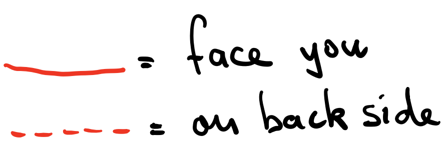

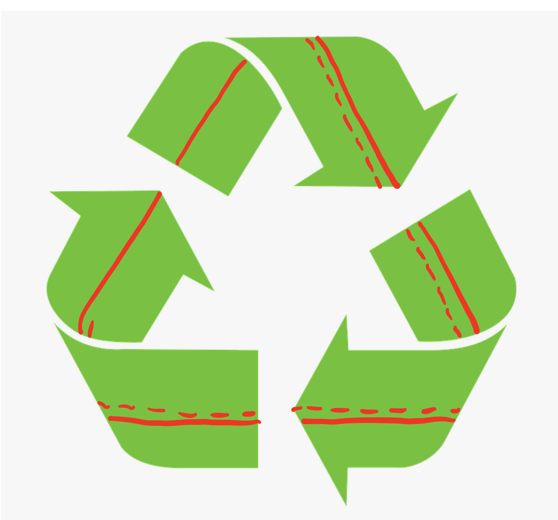

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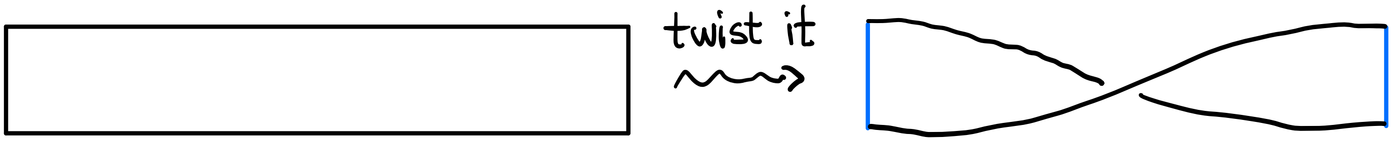

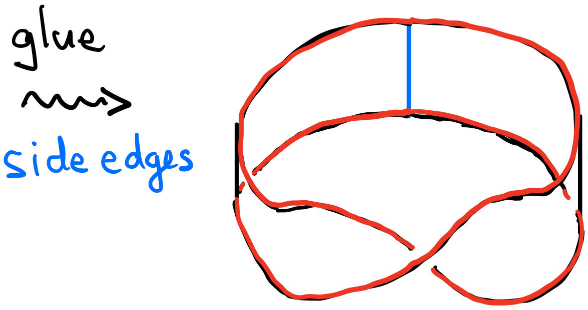

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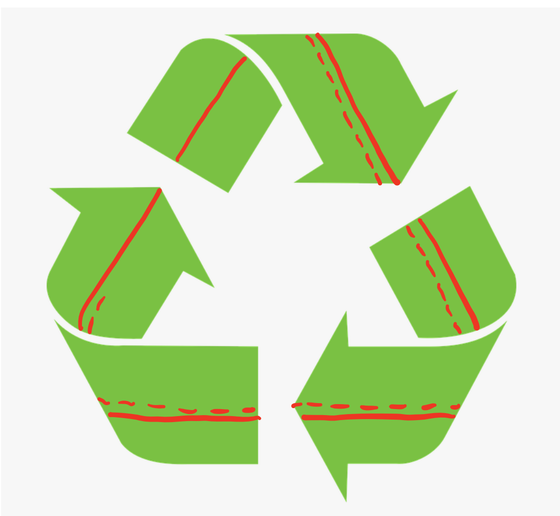
——— = face you
- - - - - = on back side



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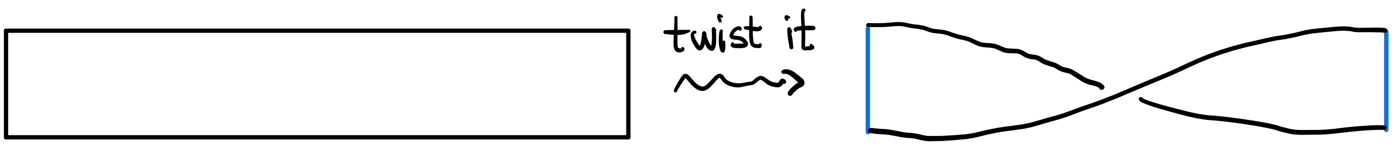

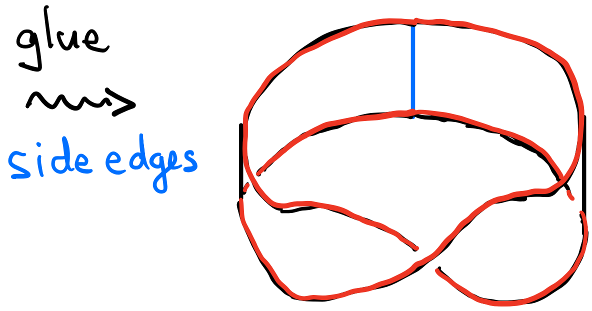

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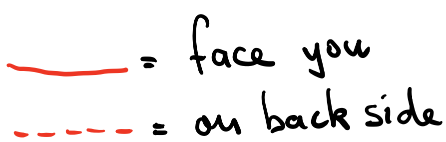

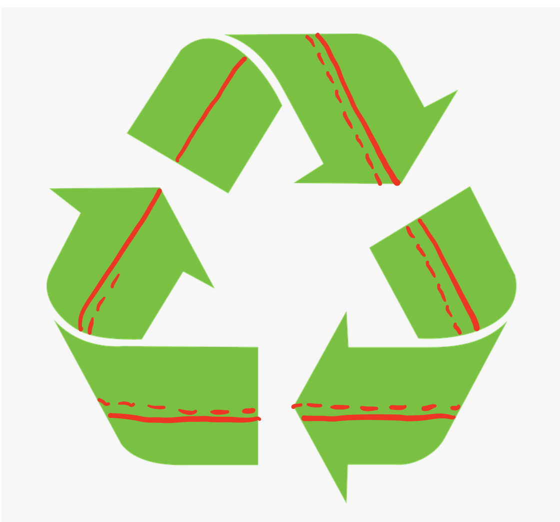

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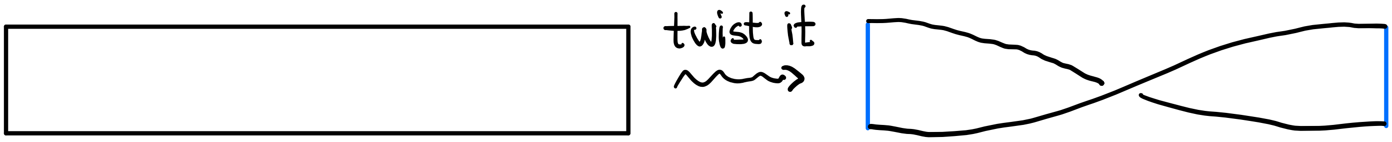

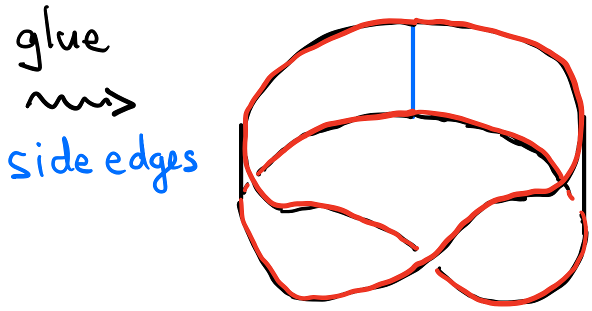

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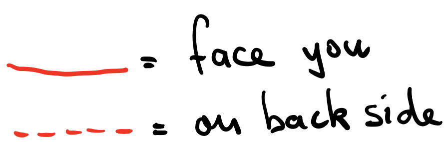

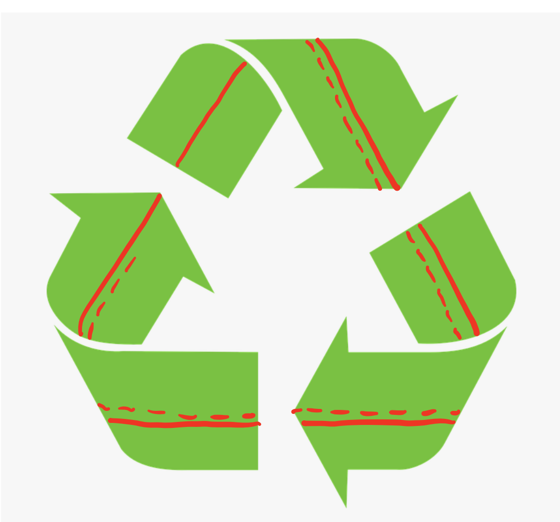

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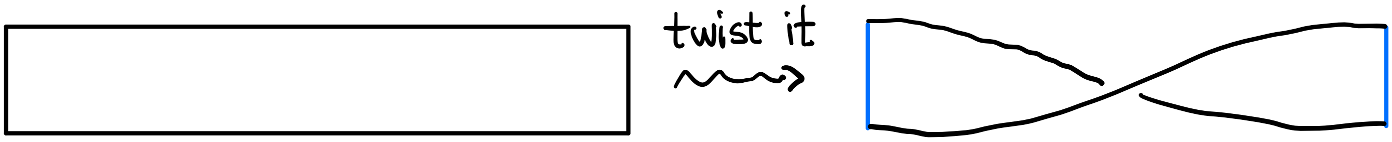

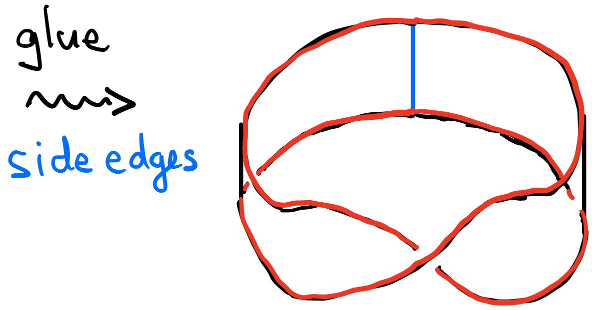

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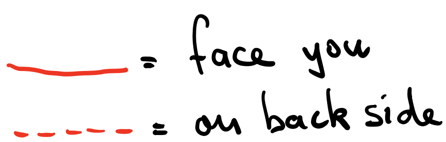

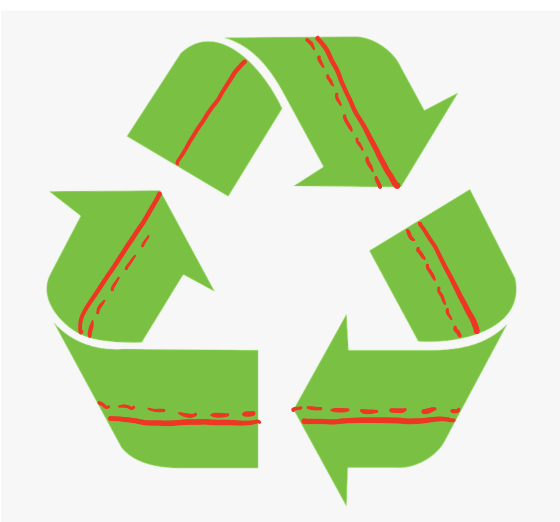

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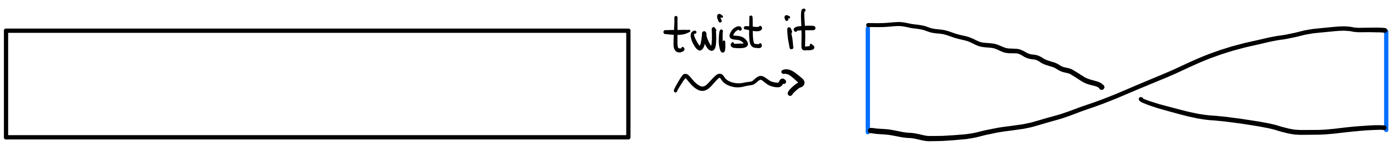

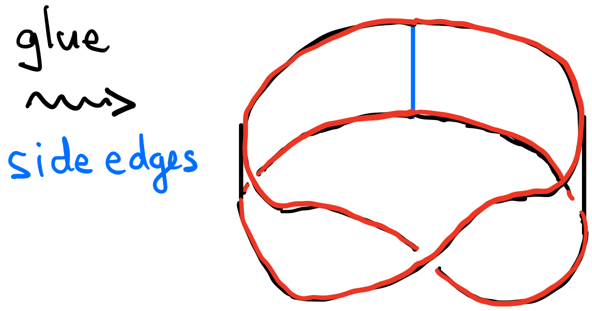

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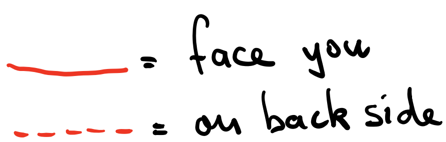

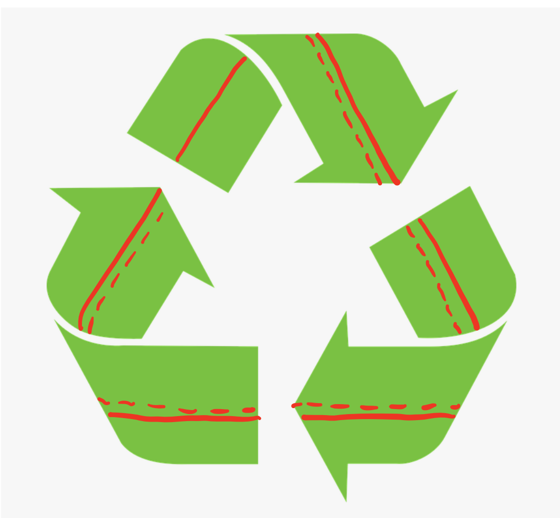

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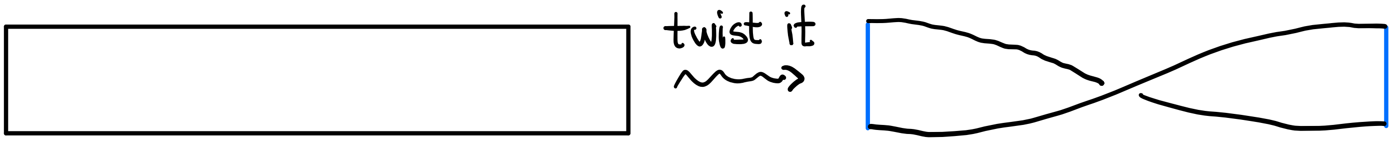

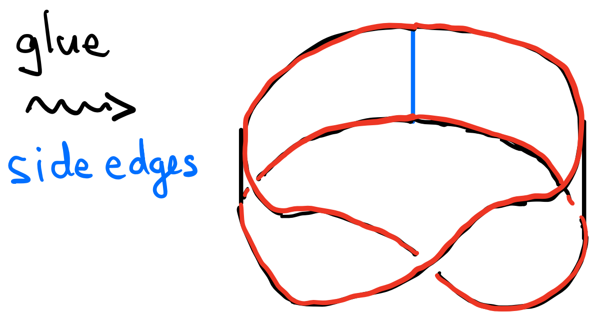

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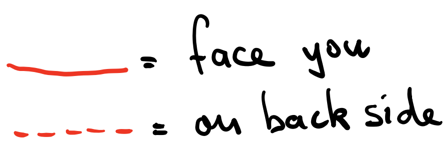

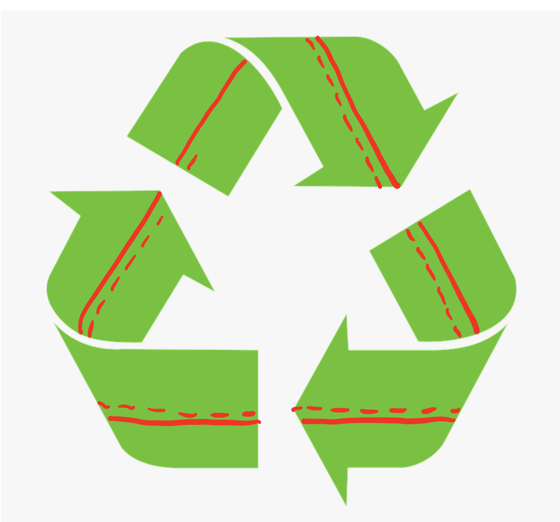

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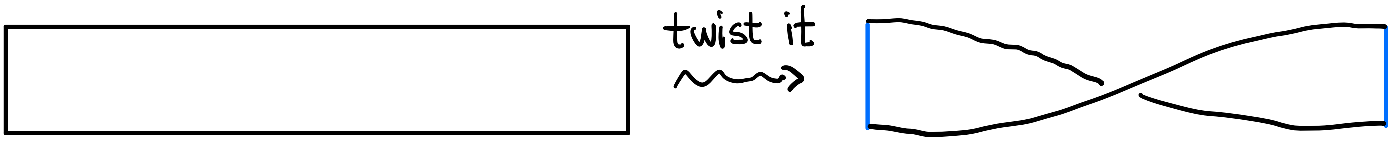

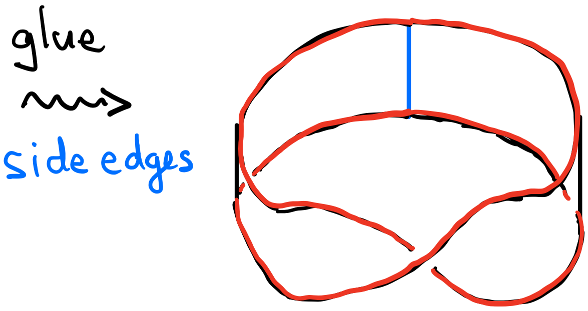

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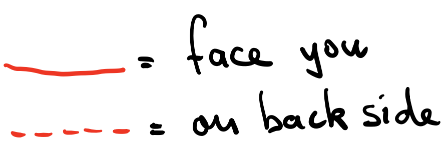

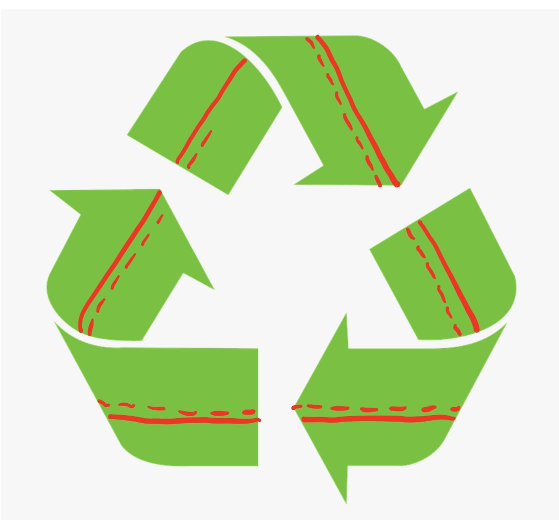

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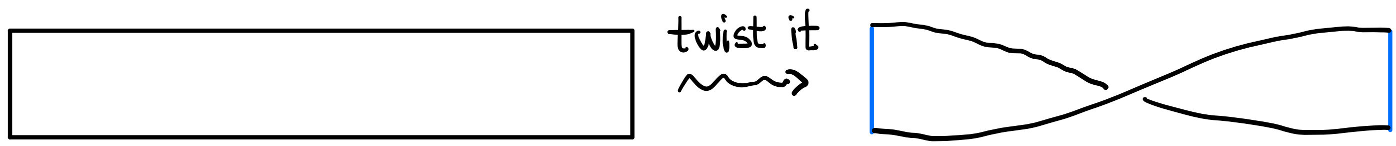

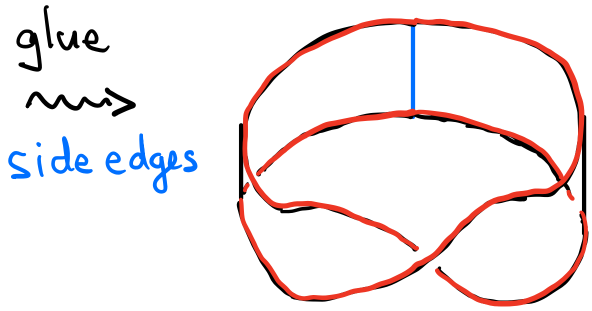

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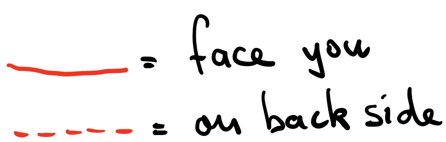

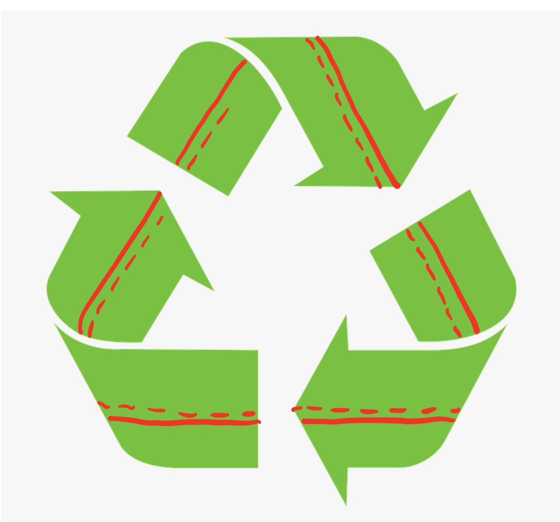

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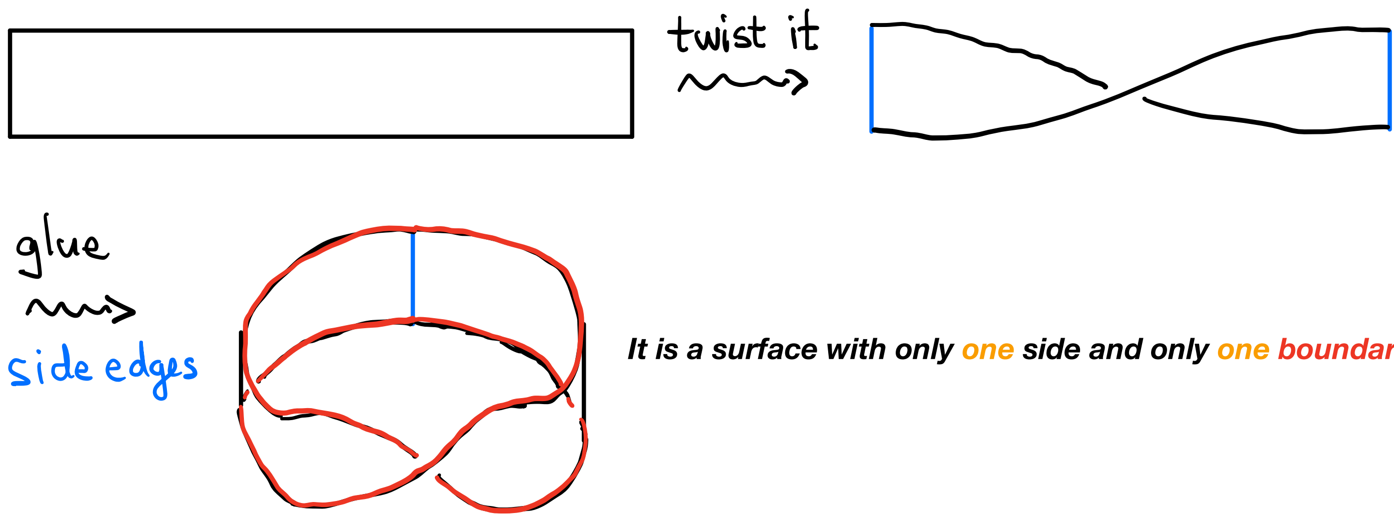

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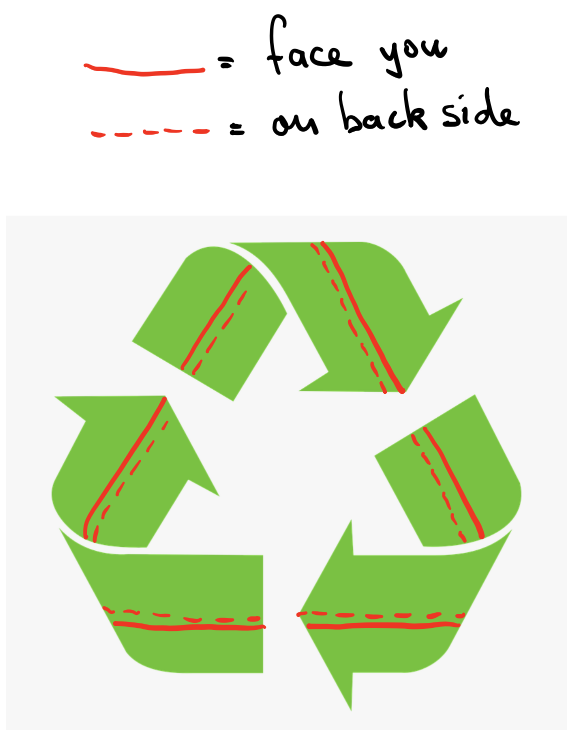

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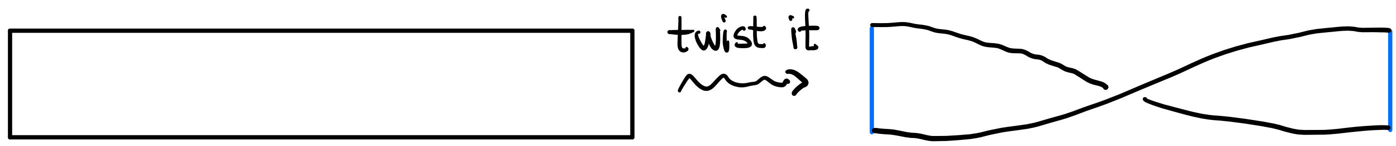

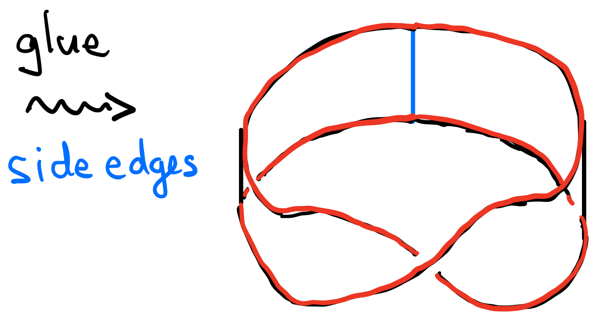

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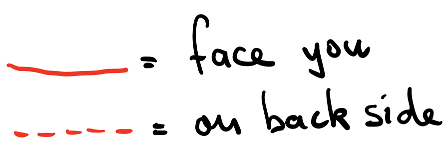

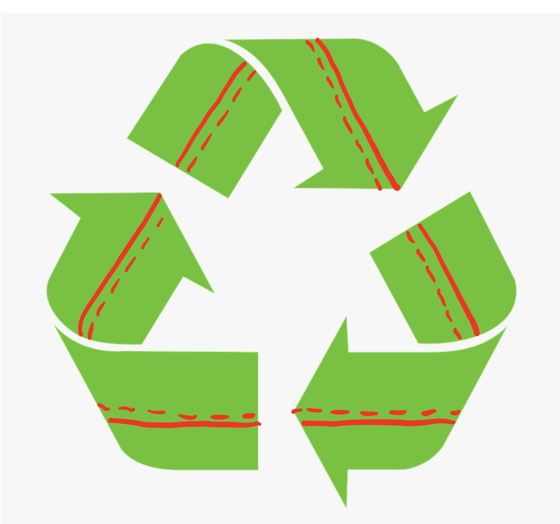

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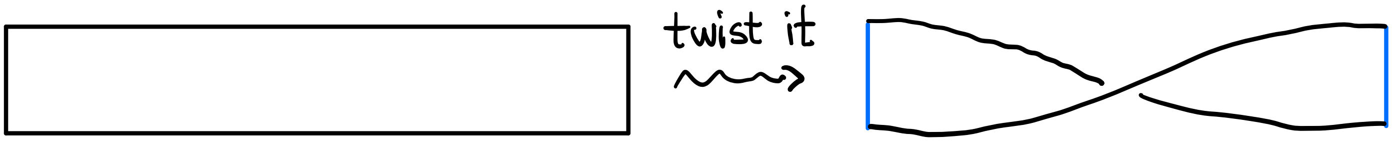
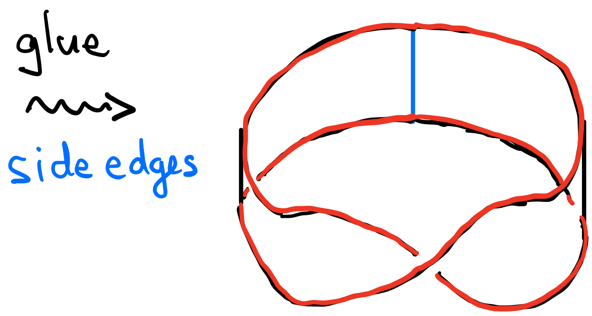
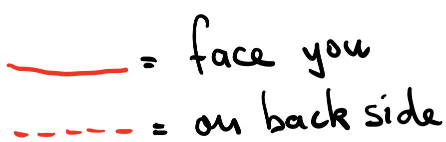


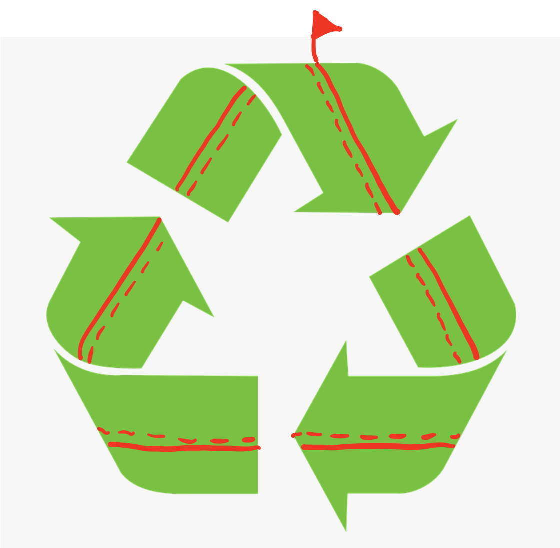


It is a surface with only one side and only one boundary!

Where do we SEE a Möbius band?

Let's RECYCLE!





The Mobius loop is the unfamiliar name of a very familiar symbol: a triangle composed of three arrows looping back on themselves in clockwise direction

This symbol indicates that a product can be recycled, but not necessarily that it has been itself produced from recycled materials

Are these really Möbius bands?

“成都高新区五岔子大桥的网红之路”

四川省建筑设计研究院，2020

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“它的设计创意来自于‘无限之环’—莫比乌斯环的概念，把四维空间中才存在的无限形态，抽象设计到三维空间中，形成了数学中无穷大的符号形象，所以可以说这个形象代表着桥梁所在的高新区无限的发展可能。”

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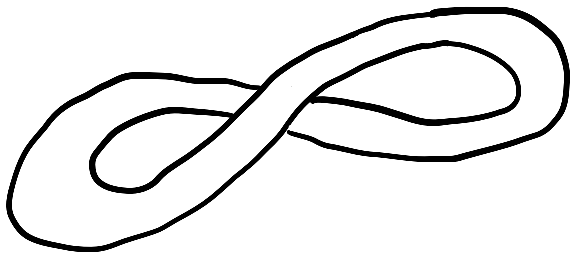

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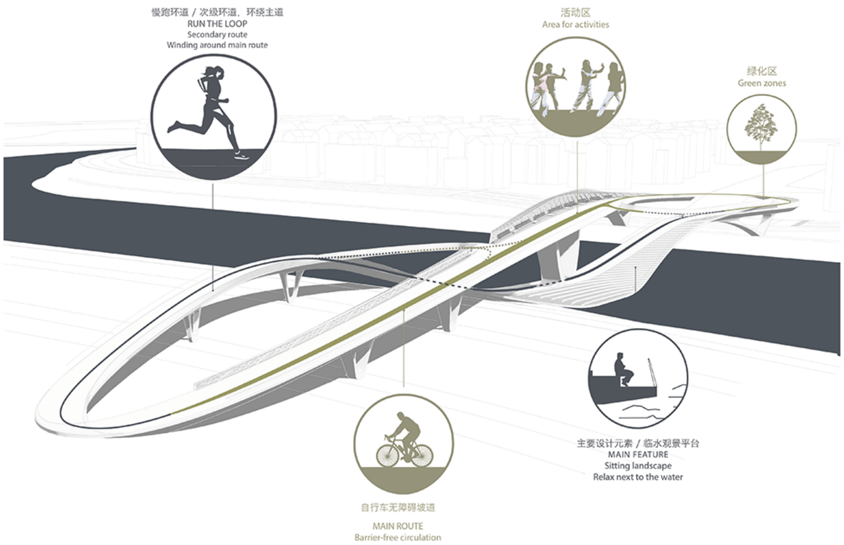

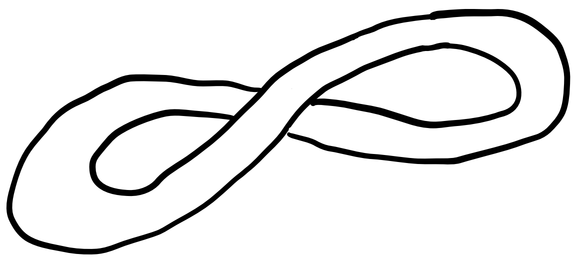

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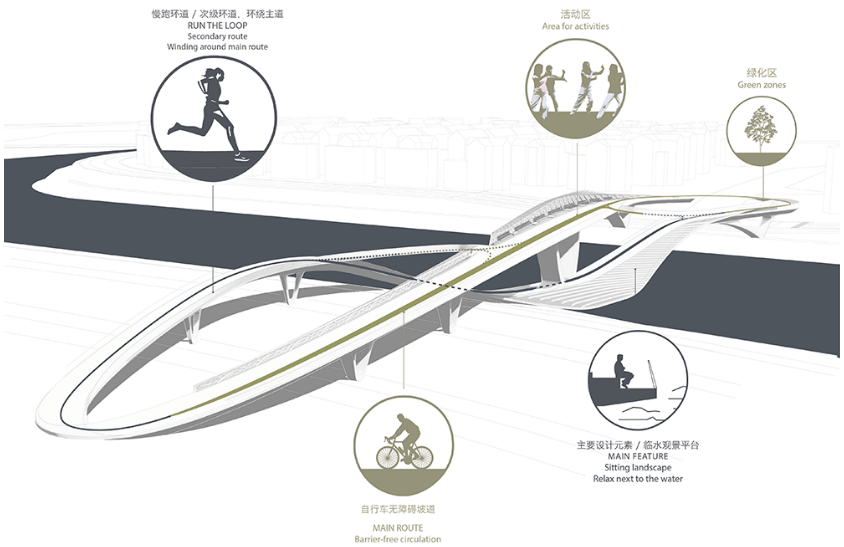

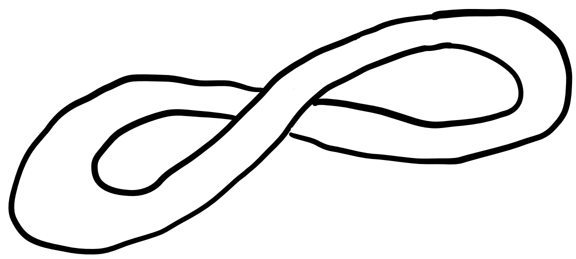

2022打卡点！

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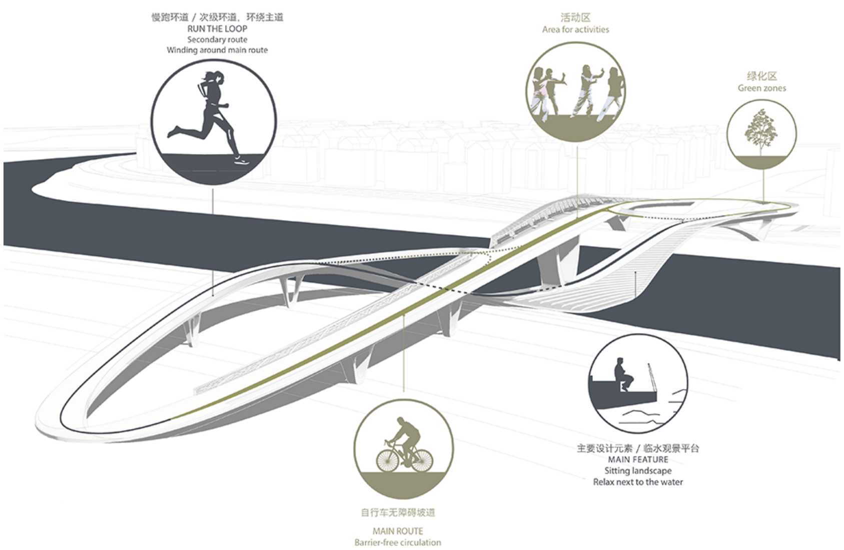

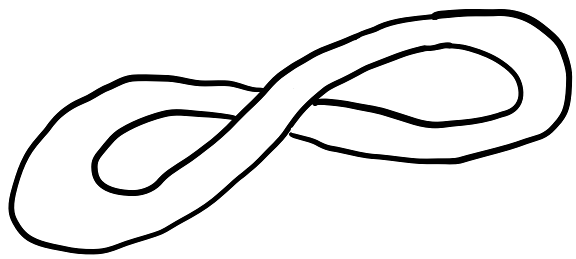

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“梅溪湖中国结桥怎么样 - 小红书”

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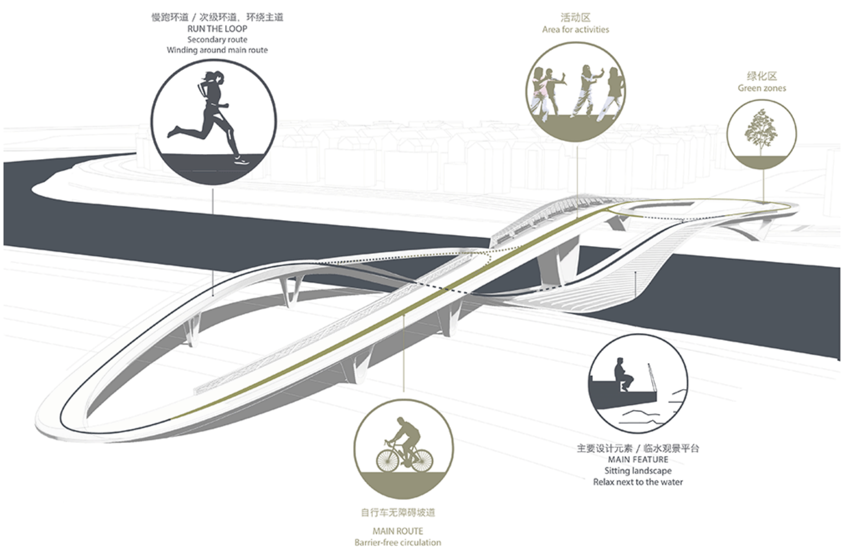

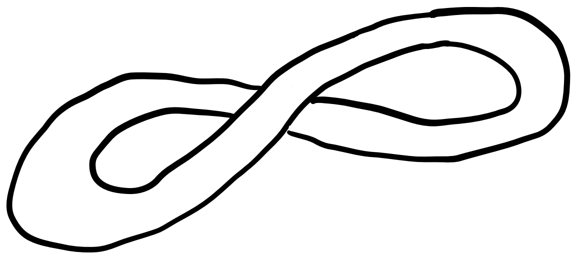

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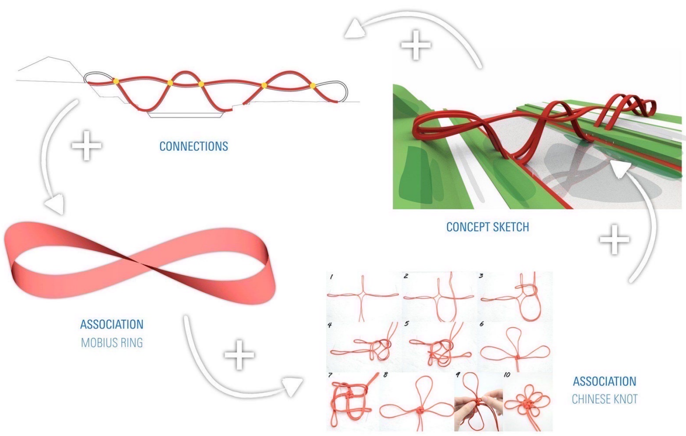

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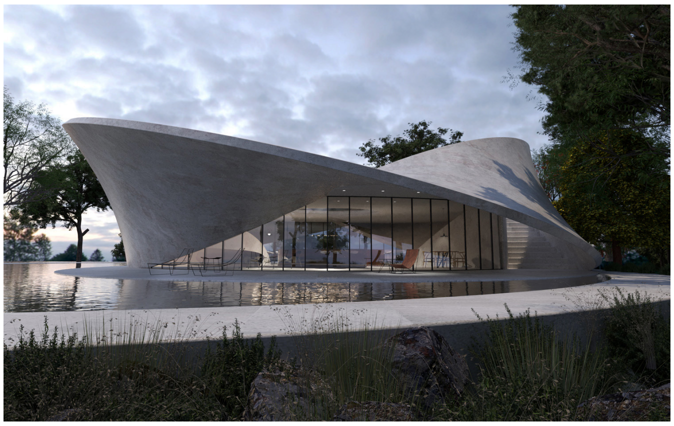
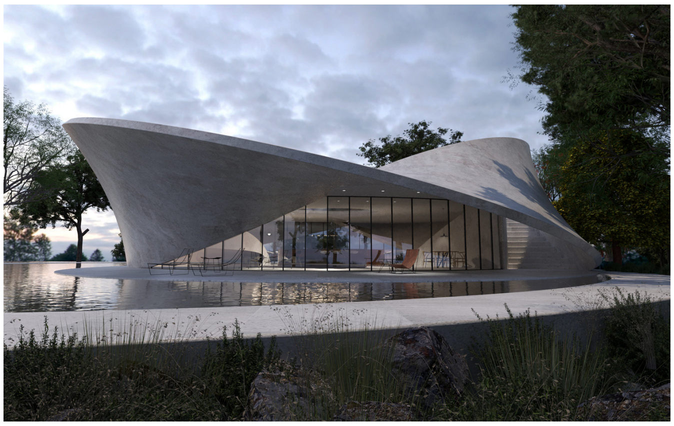
Architecture designs by Antony Gibbon



Architecture designs by Antony Gibbon

Inspired~



MOBIUS

The design evolved from the Mobius strip which is a surface with only one side and only one boundary. It has the mathematical property of being unorientable

The circular interior sits beneath the organic form. Floor to ceiling glass doors circulate the open plan living space and lead you out to the pool area. A circular kitchen is at the centre point of the Mobius house with a sky light that mirrors the diameter of the kitchen shape directly above. A twisted staircase leads you up onto the roof terrace that follows the form of the Hempcrete internal walls of the structure

The large roof top creates another area of equal size to the interior space providing many options for its use as well as an area to view the surrounding nature. A large eclipse shape swimming pool follows the form of the house, accessed from both sides of the building. The twisting driveway to the property takes you down into the garage which is situated directly below the building with a second staircase that takes you back up to the main interior

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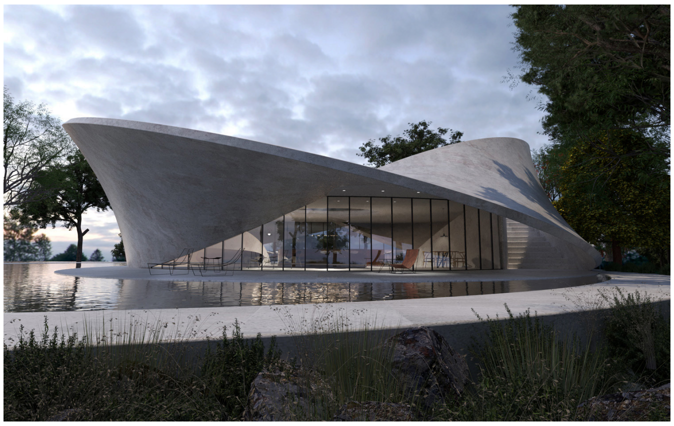

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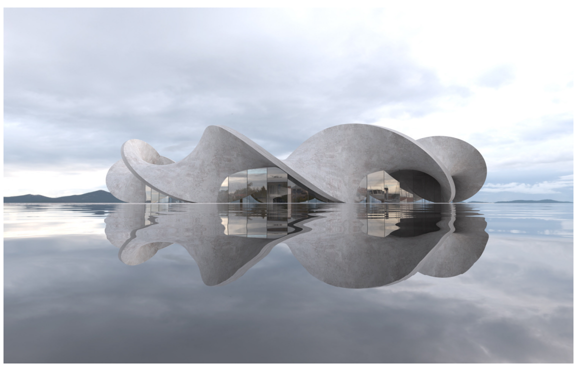
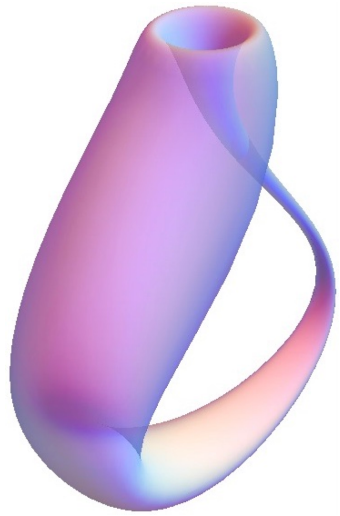
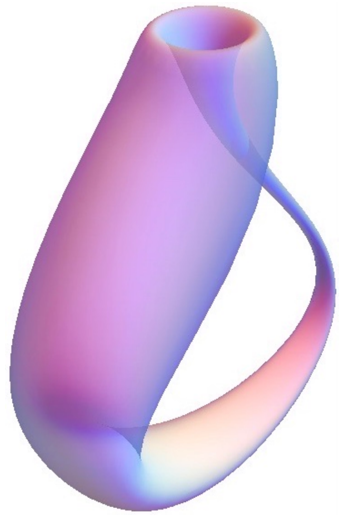


TENDRIL GALLERY

Klein bottle



Klein bottle = two Möbius bands glued together!



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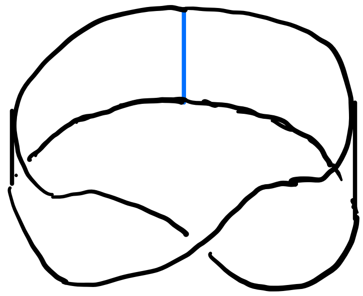

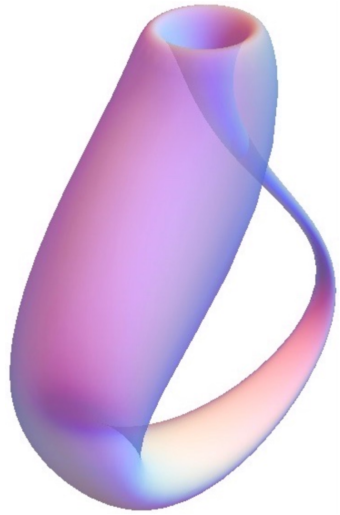

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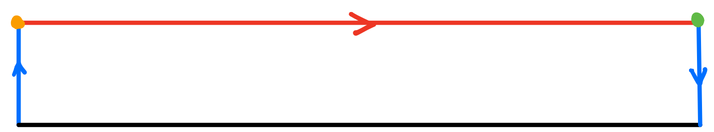

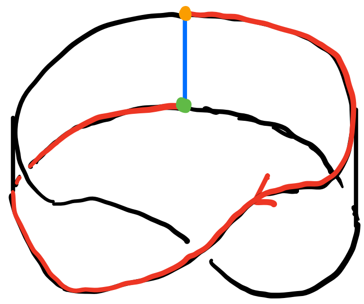

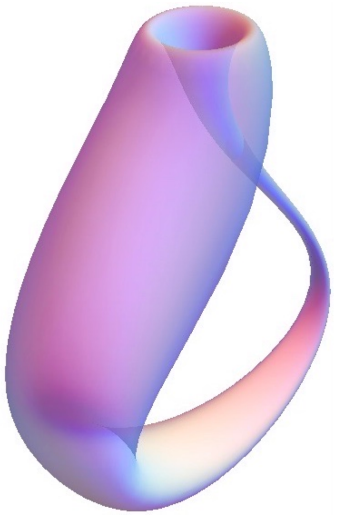

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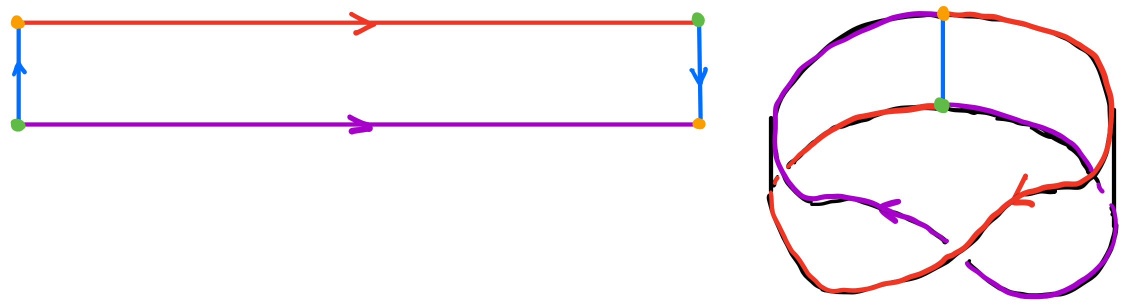

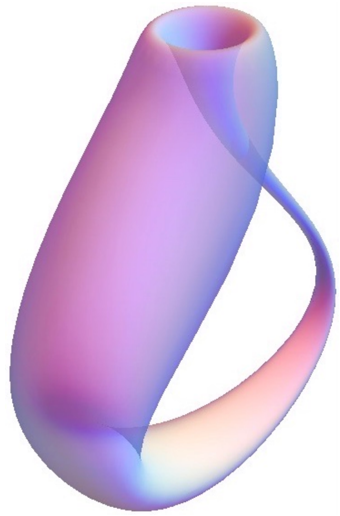

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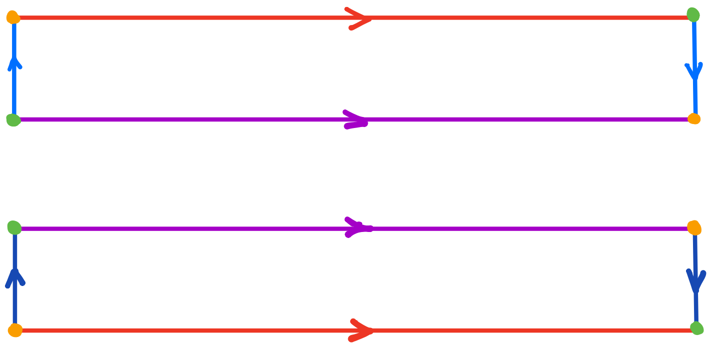

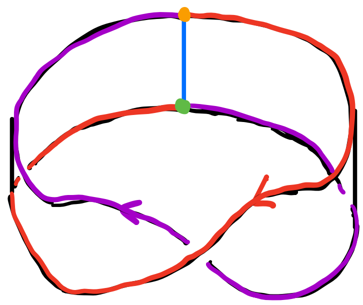

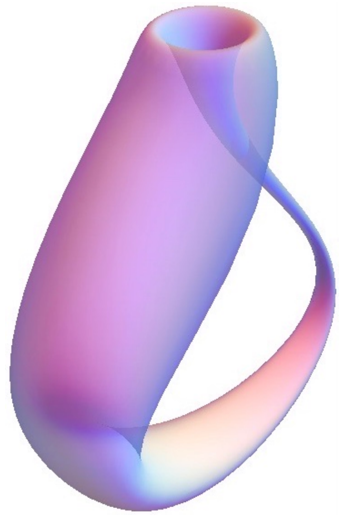

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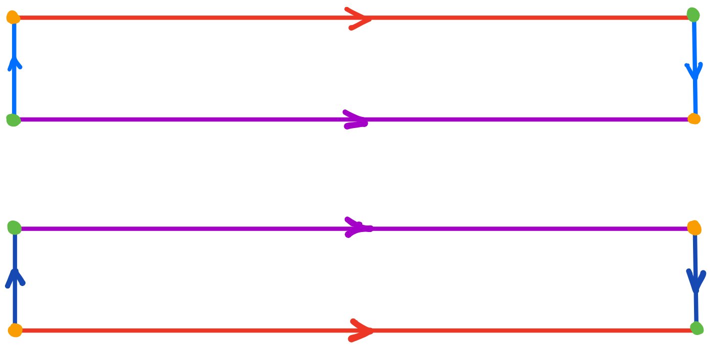
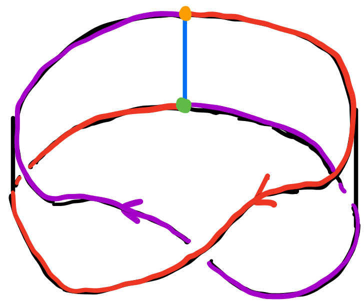
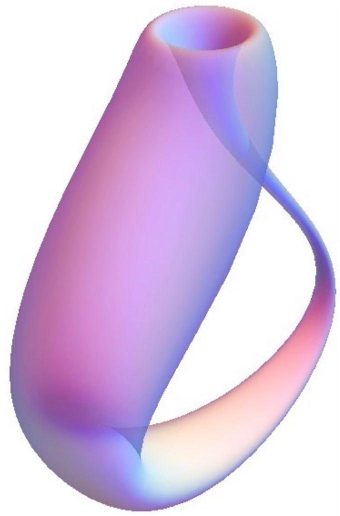


glue along } purple edges





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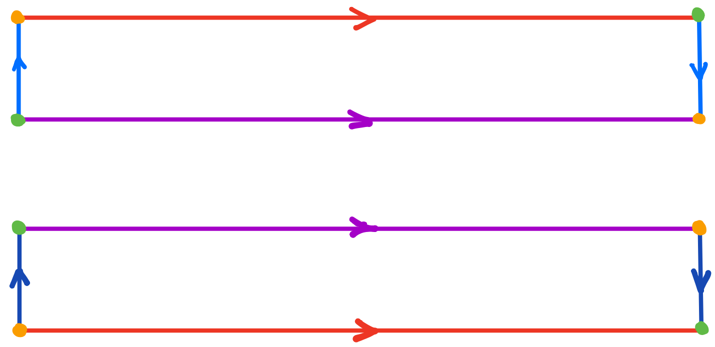

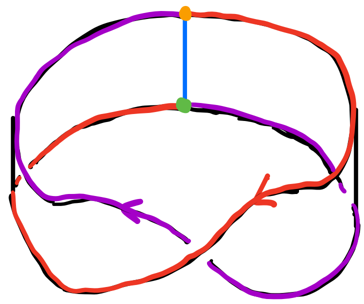

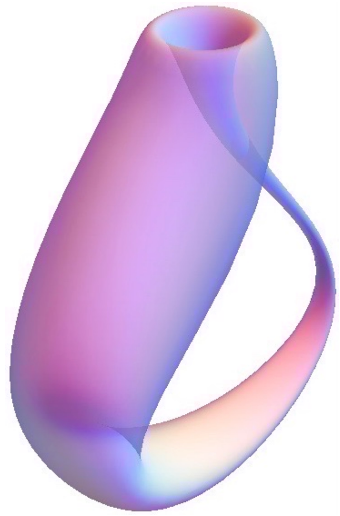

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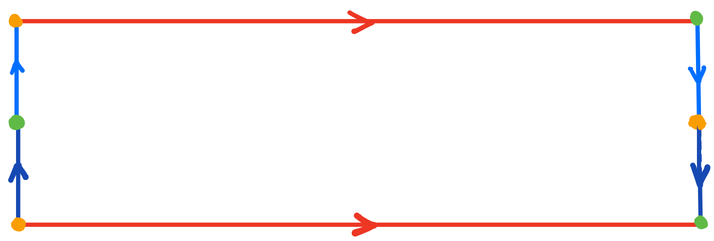

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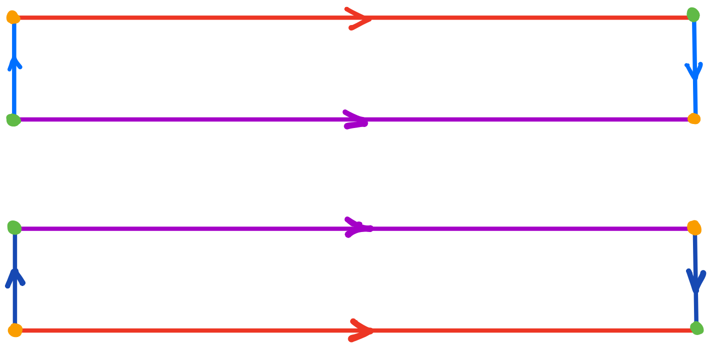
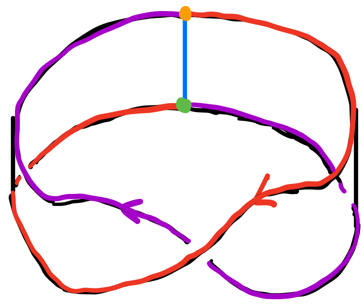
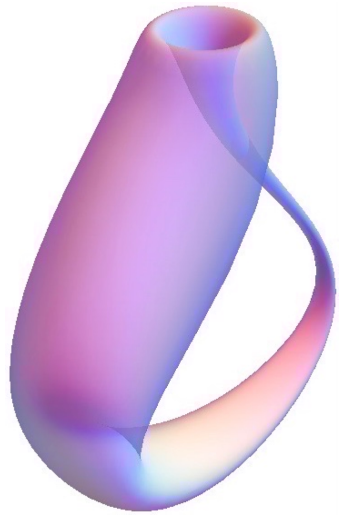
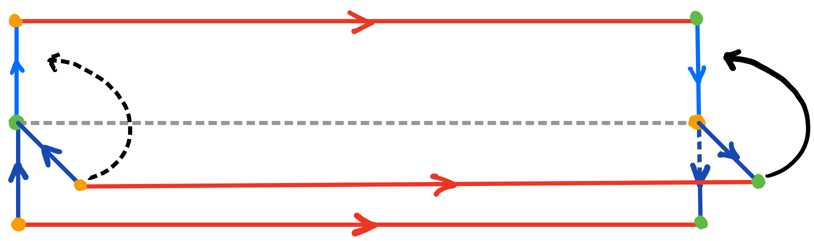






glue along } purple edges



glue along } red edges

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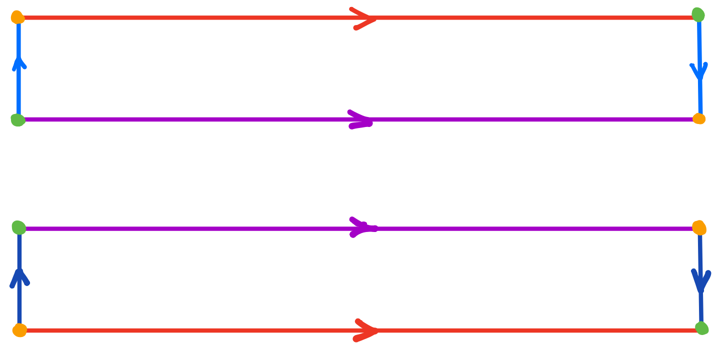

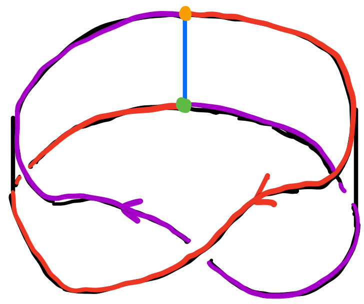

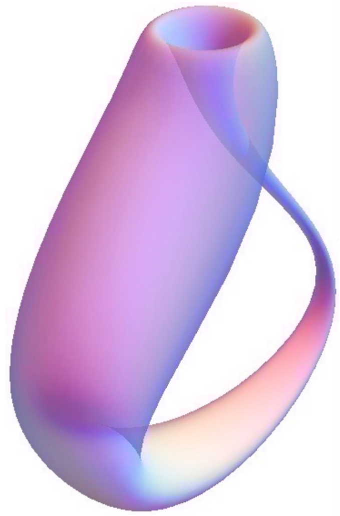

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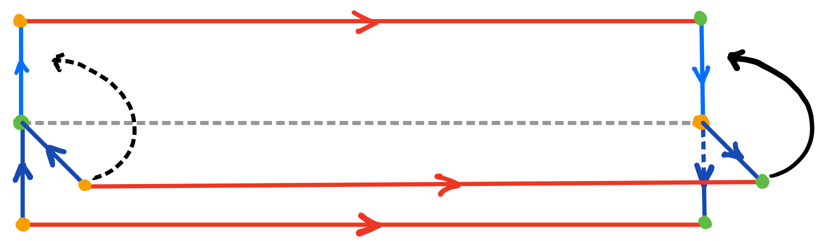

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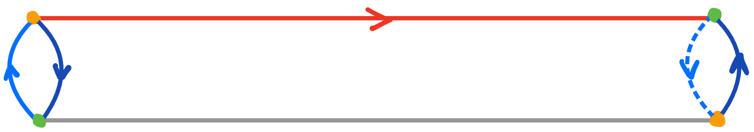

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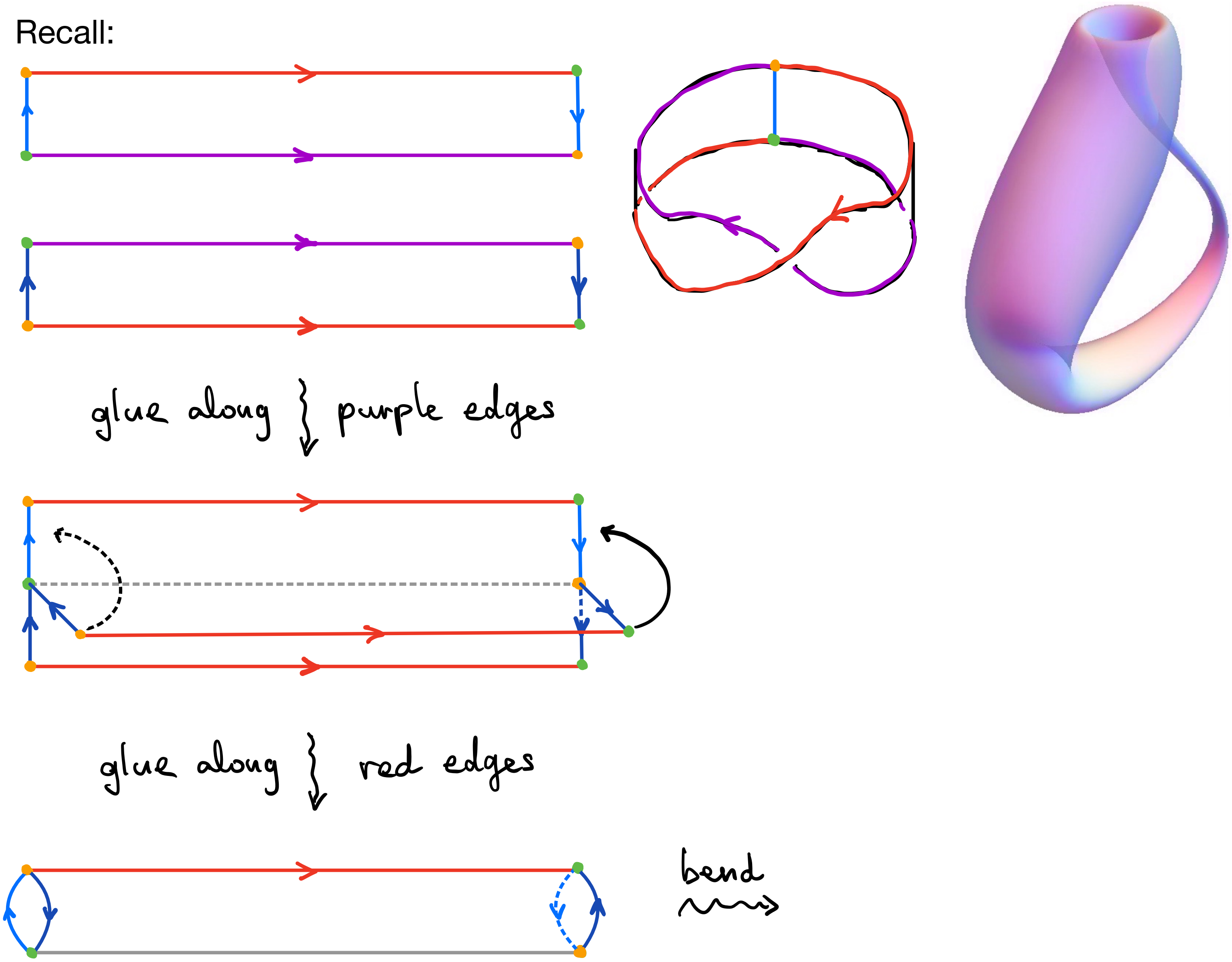

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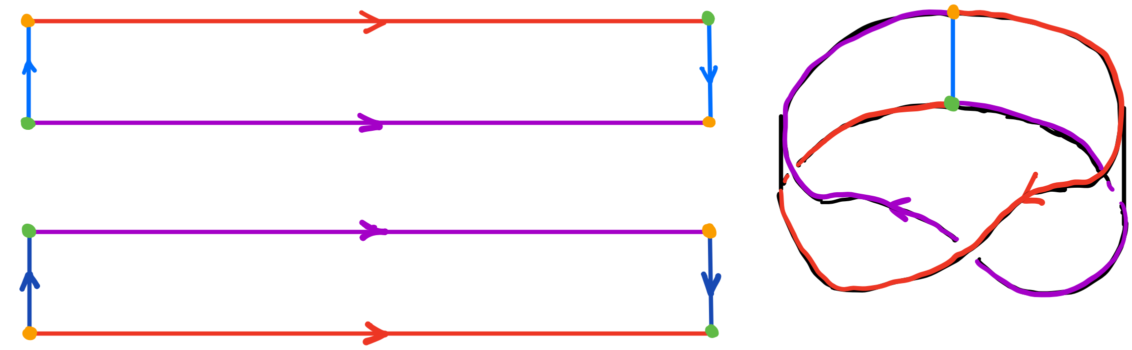

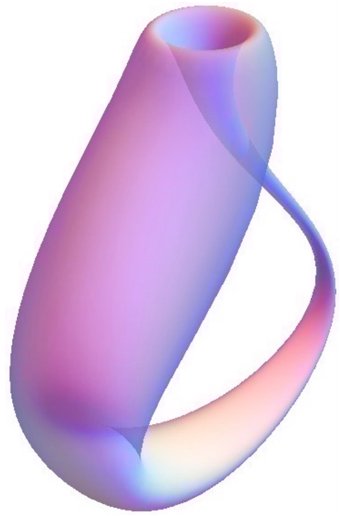

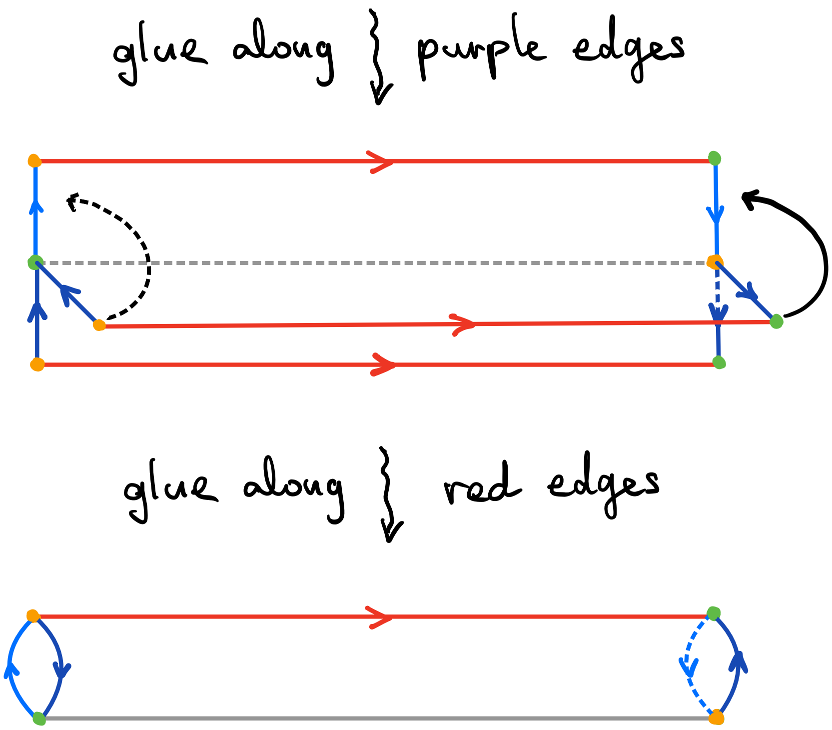

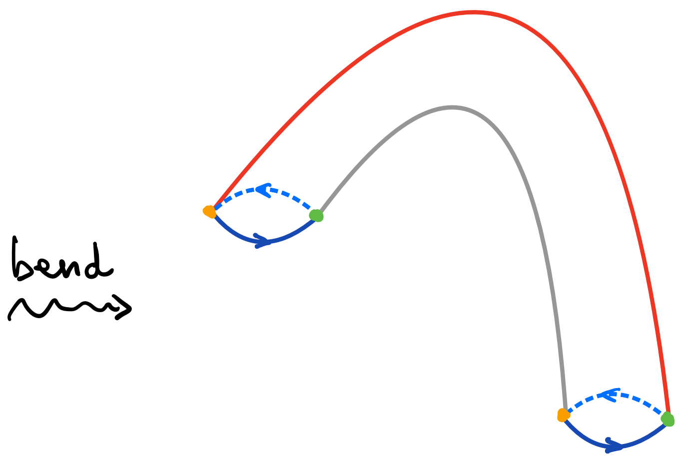

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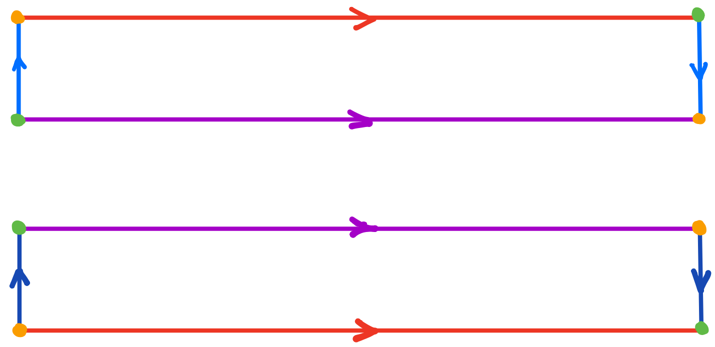
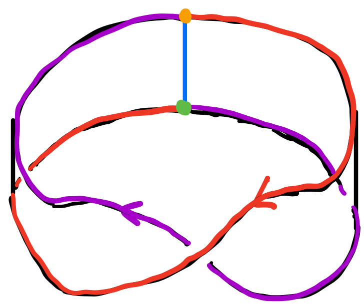
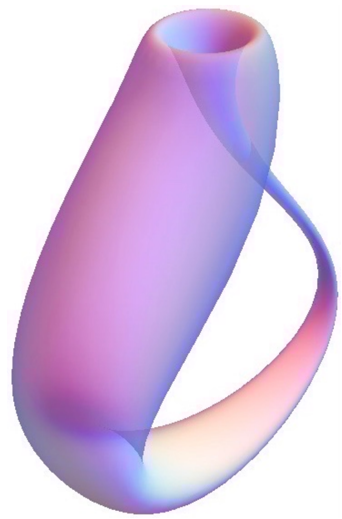
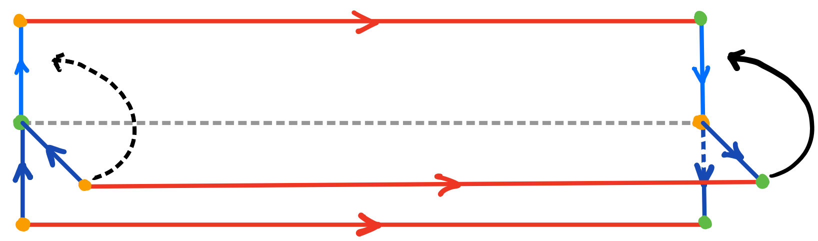


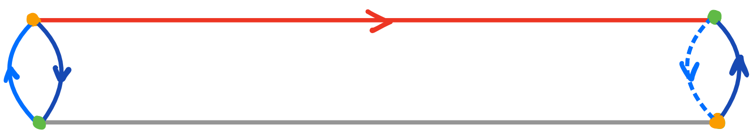




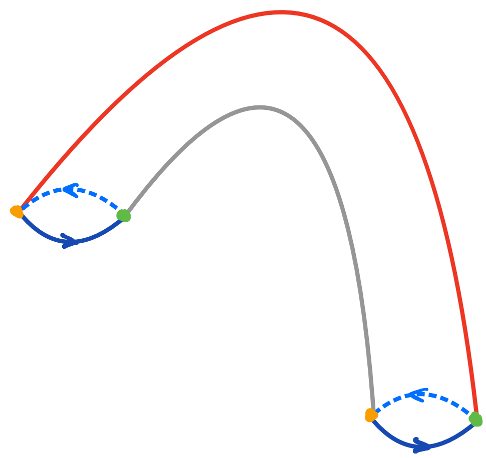
glue along purple edges



glue along red edges



bend



bend further

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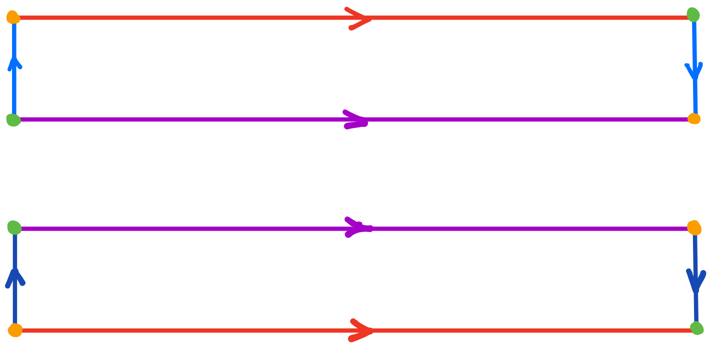
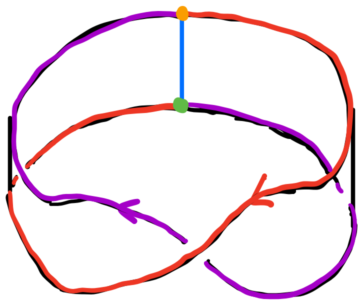
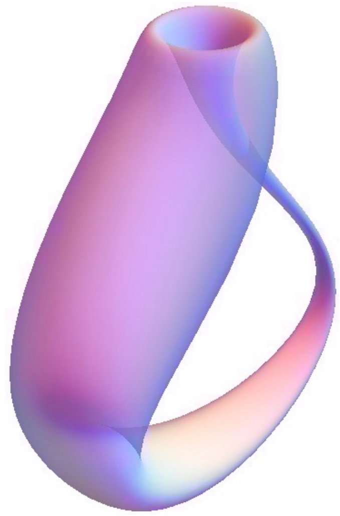
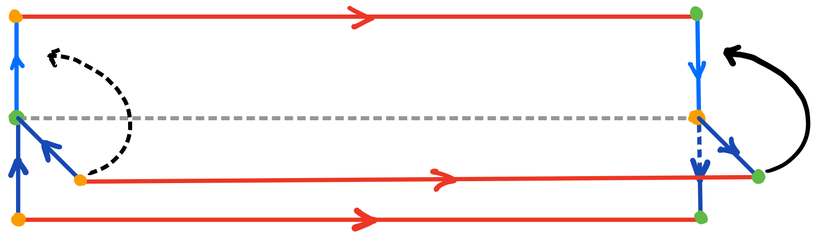
Recall:



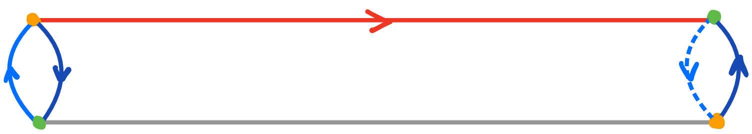




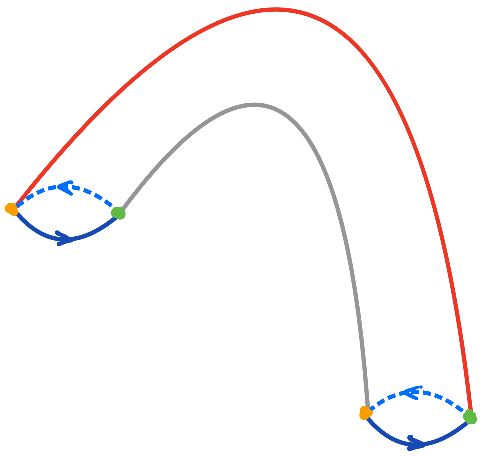
glue along ⌄ purple edges



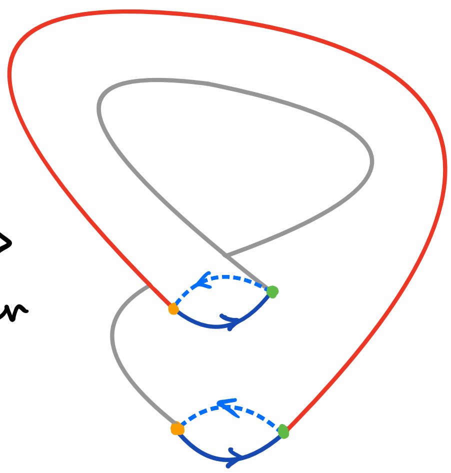
glue along ⌄ red edges



bend ⇝



bend ⇝ further



Klein bottle = two Möbius bands glued together!

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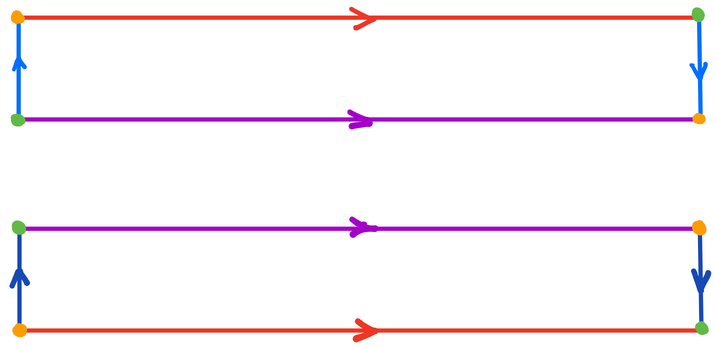
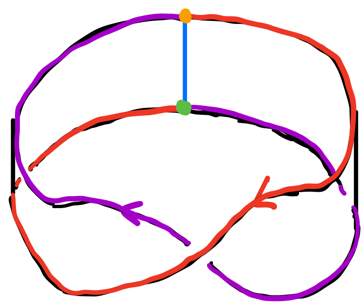
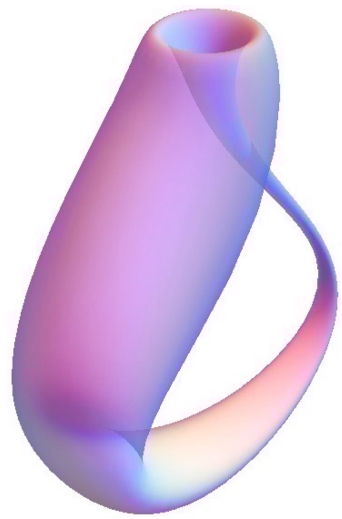
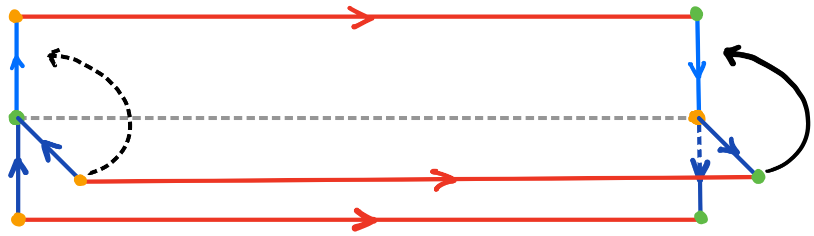


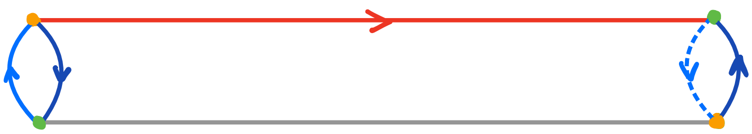




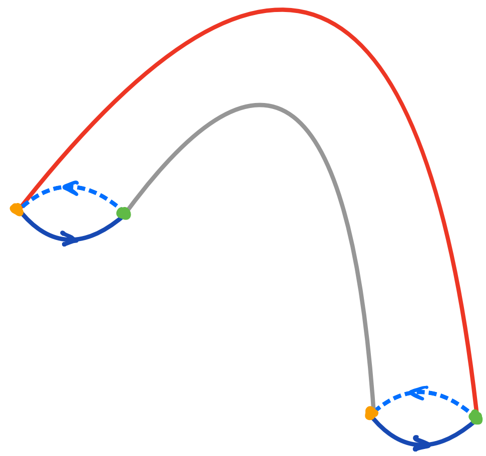
glue along ⌇ purple edges



glue along ⌇ red edges



bend ⇝



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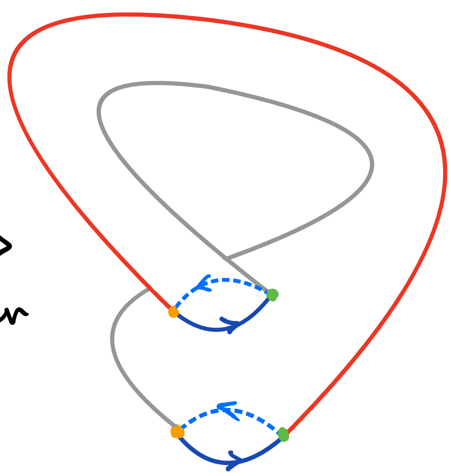

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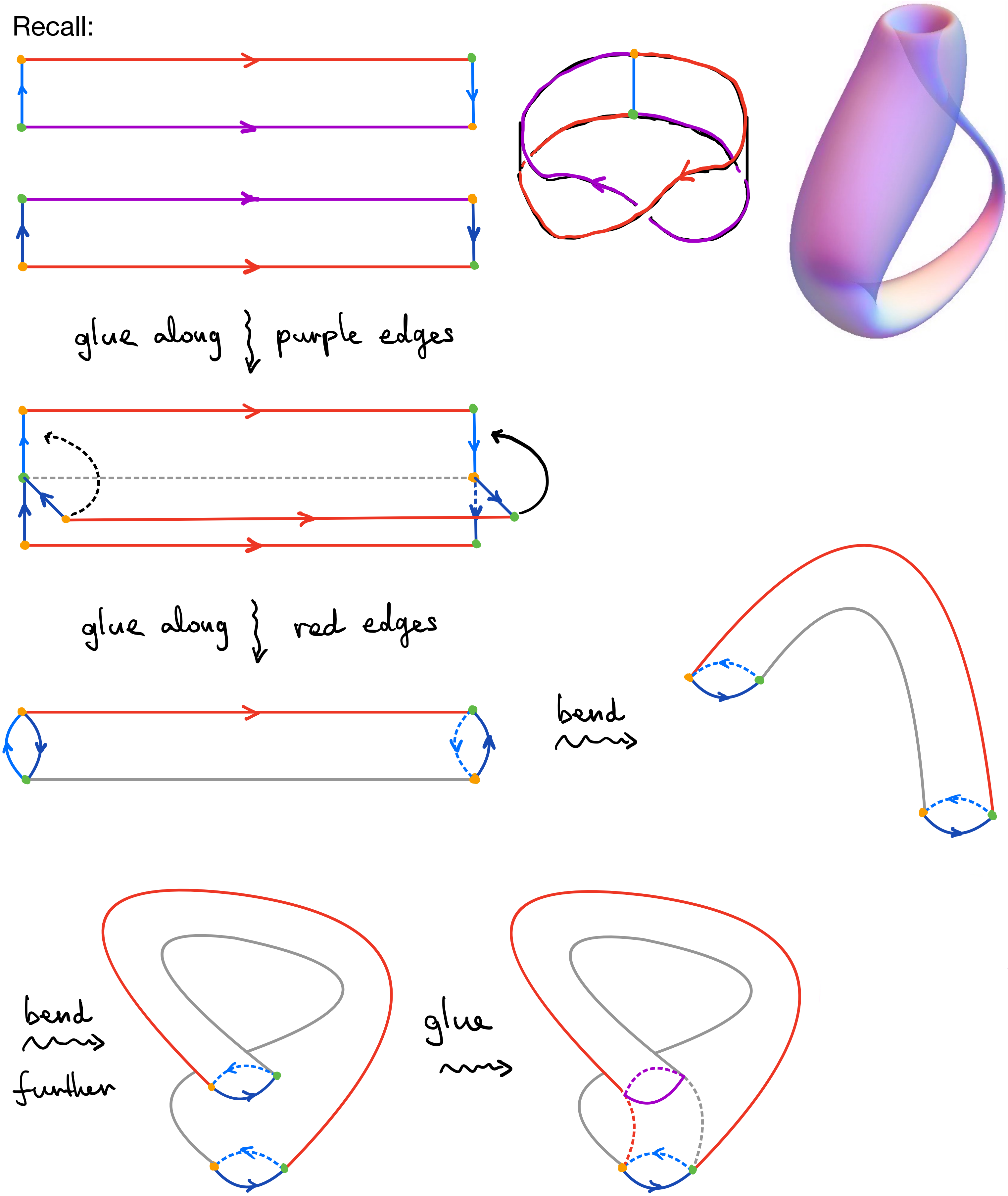

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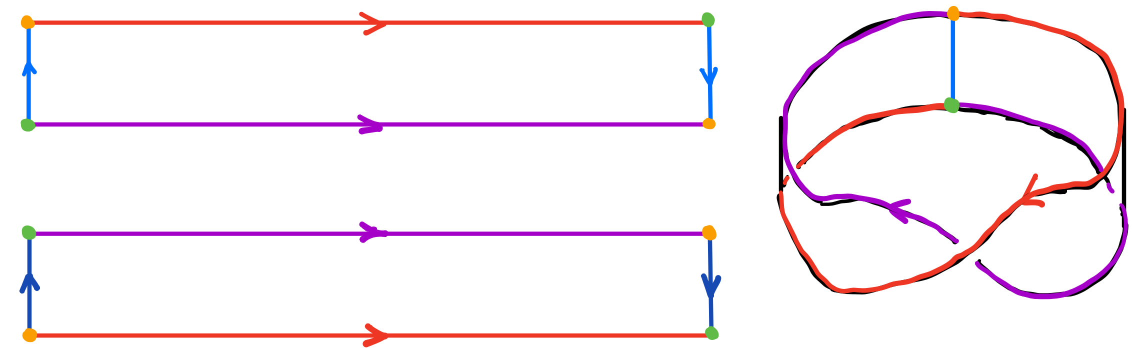

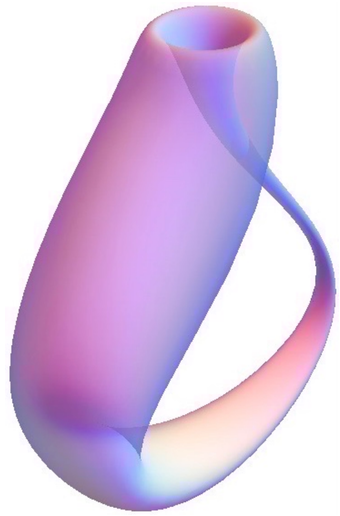

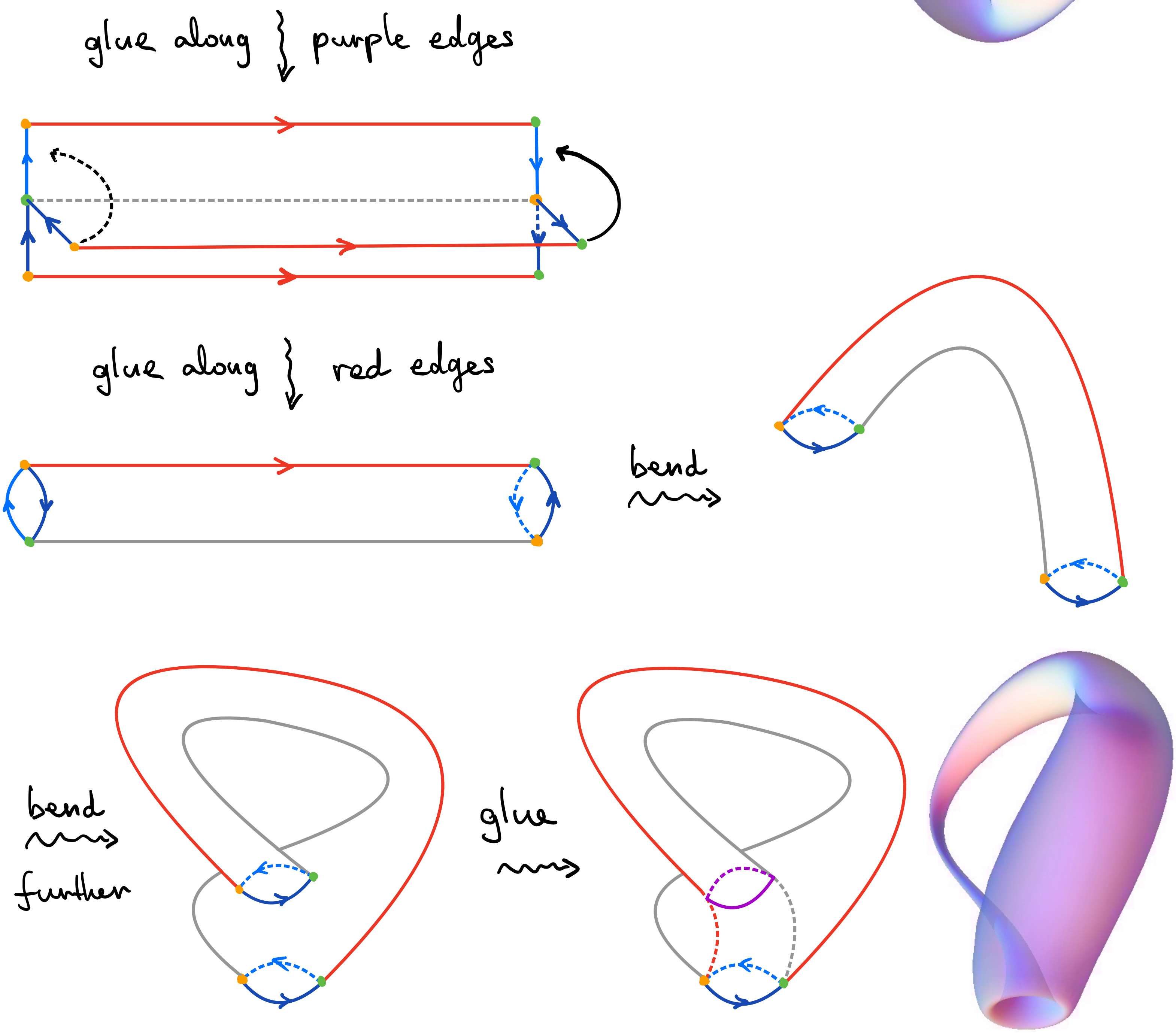

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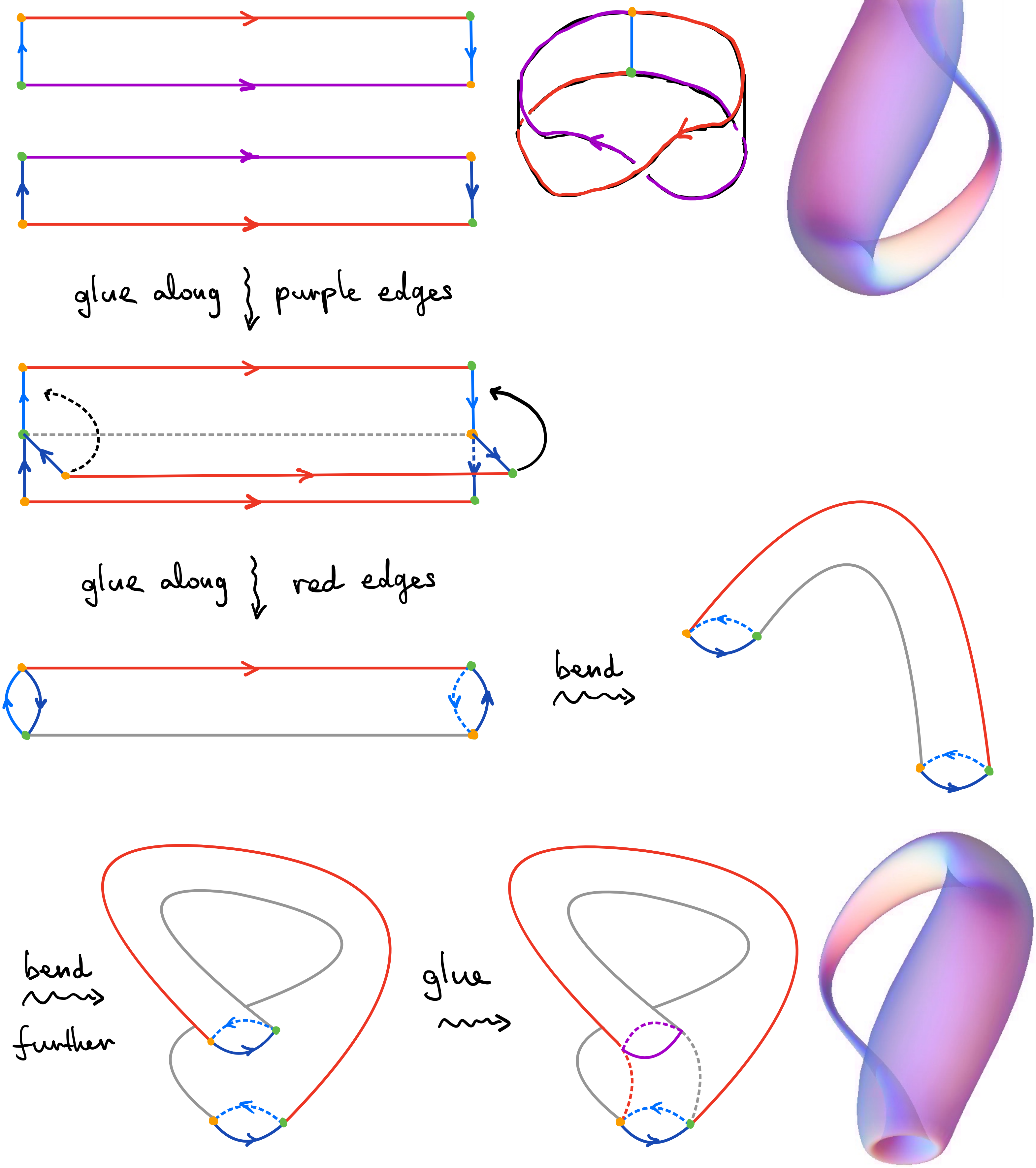

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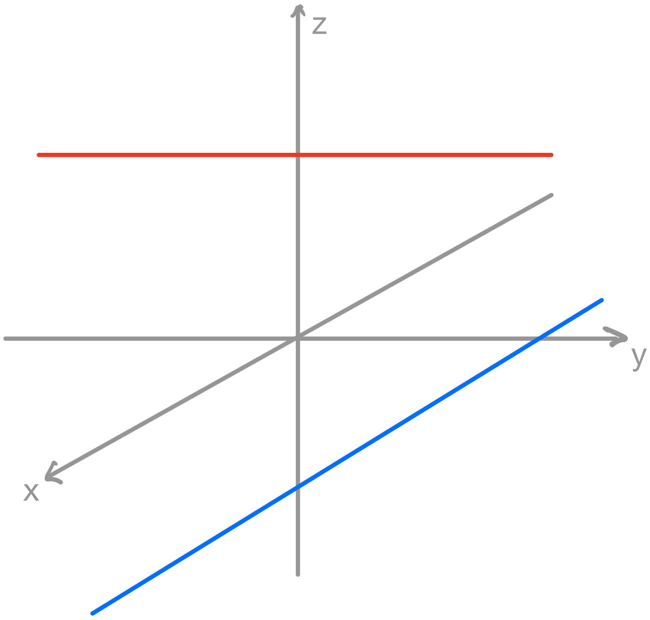

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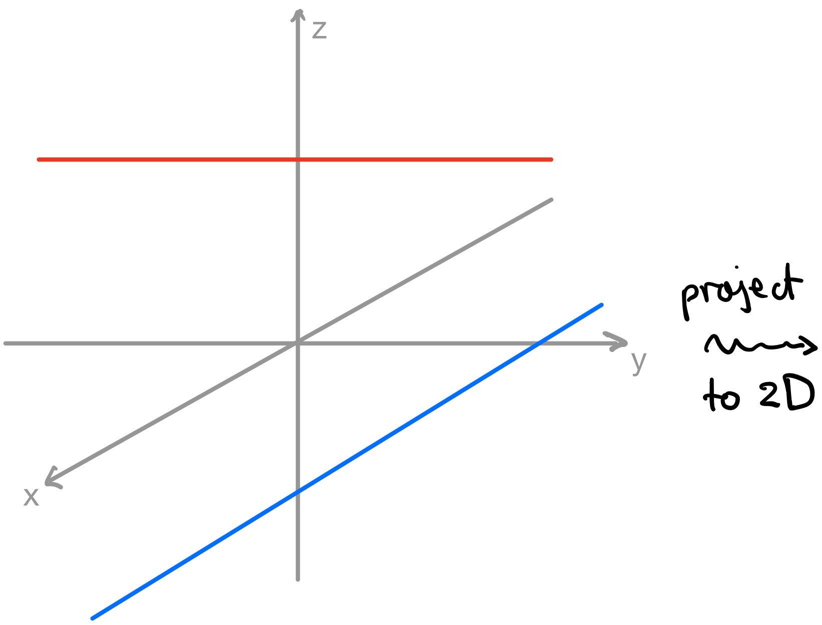

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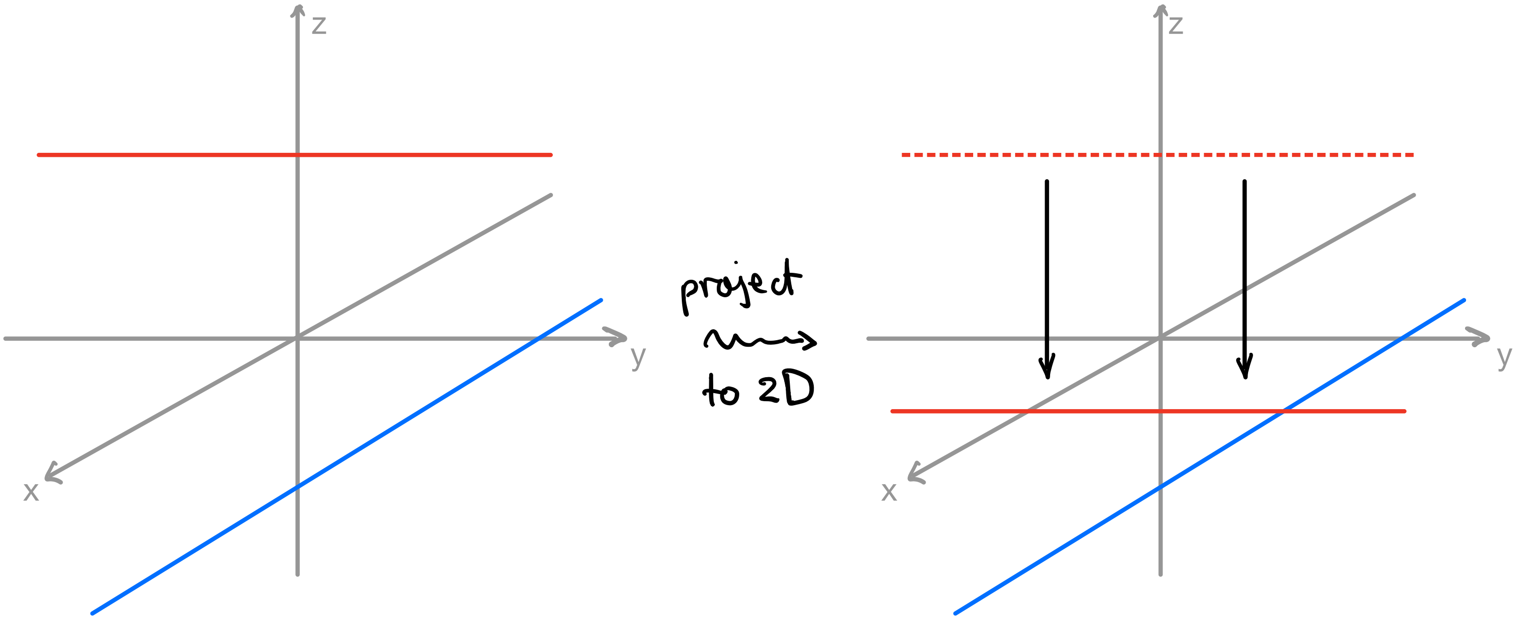

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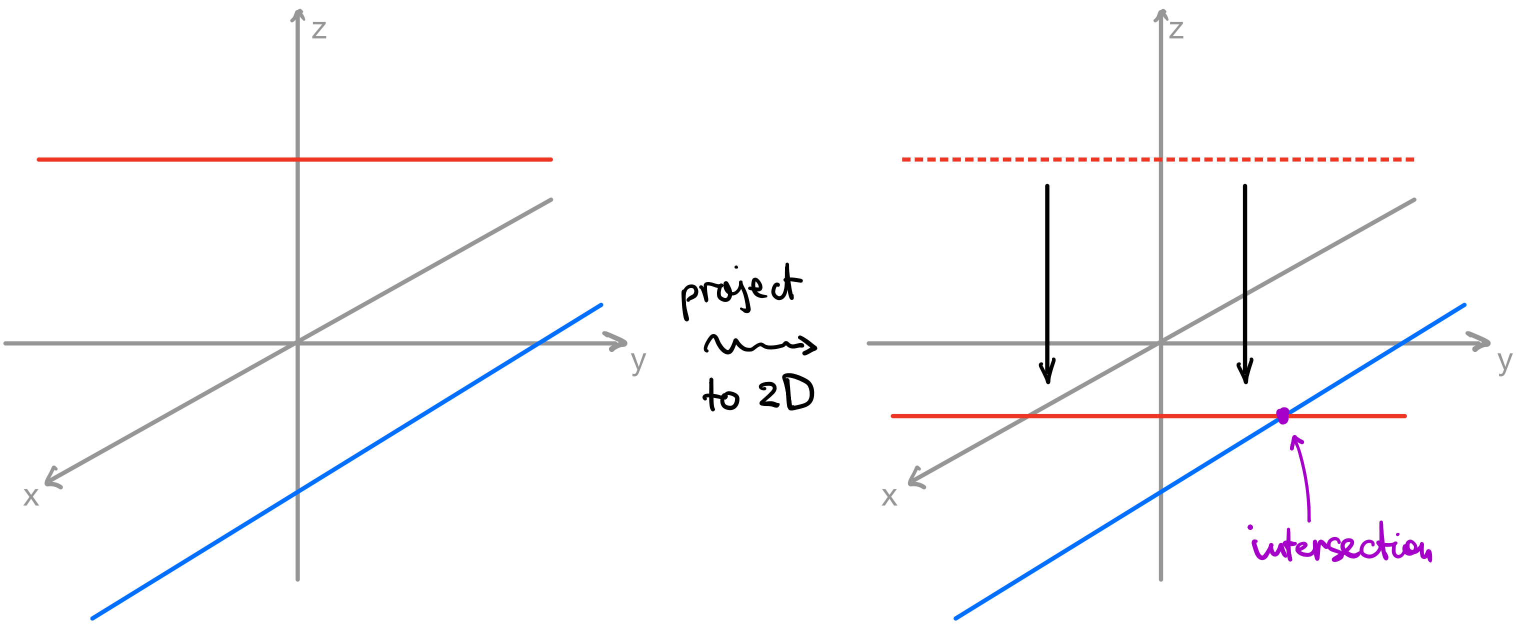

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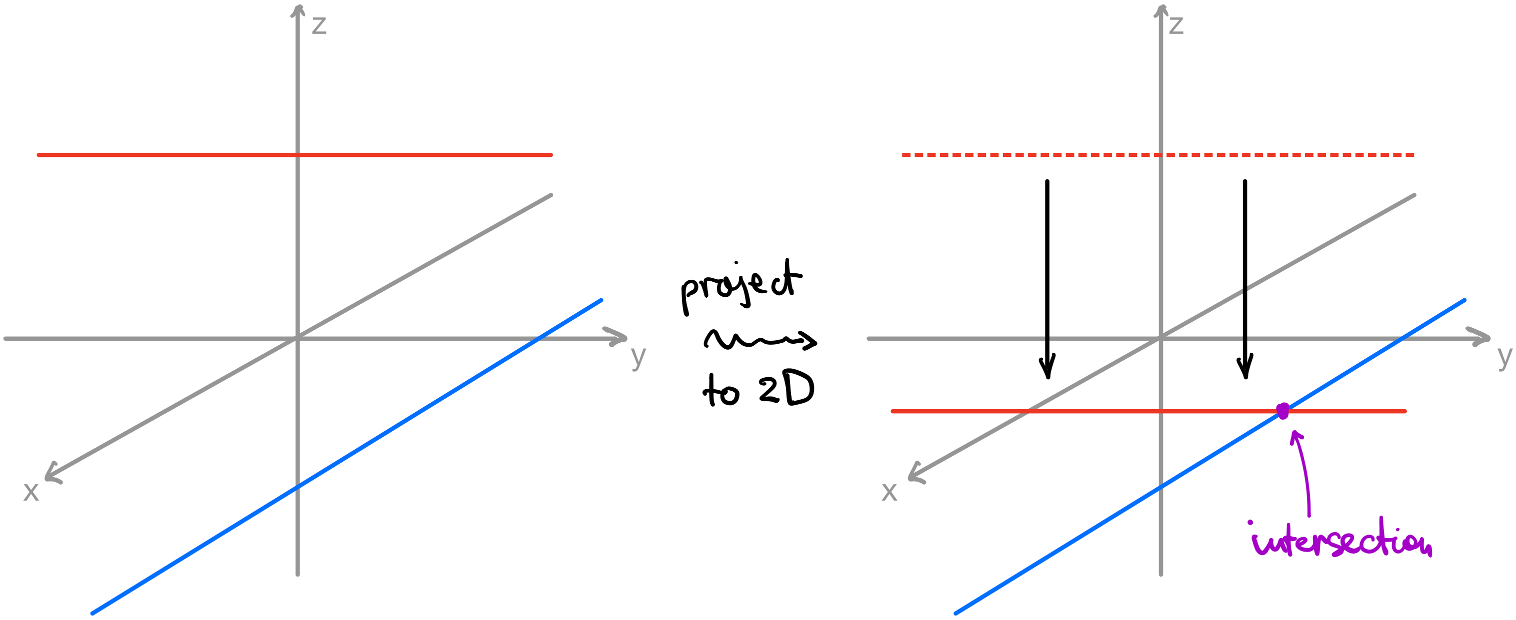

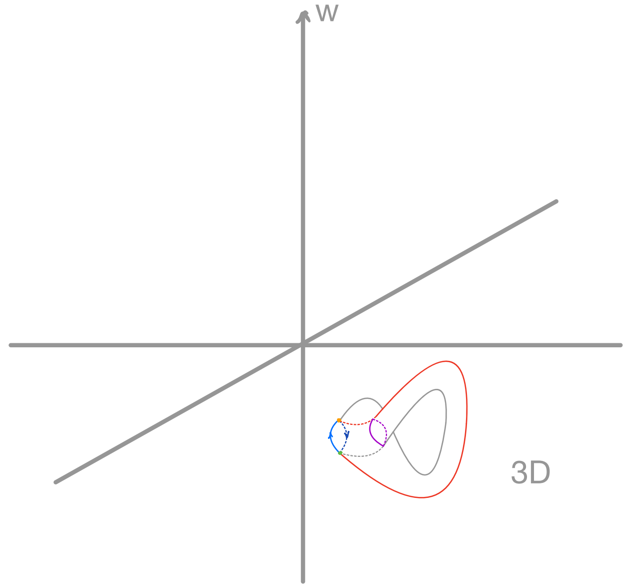

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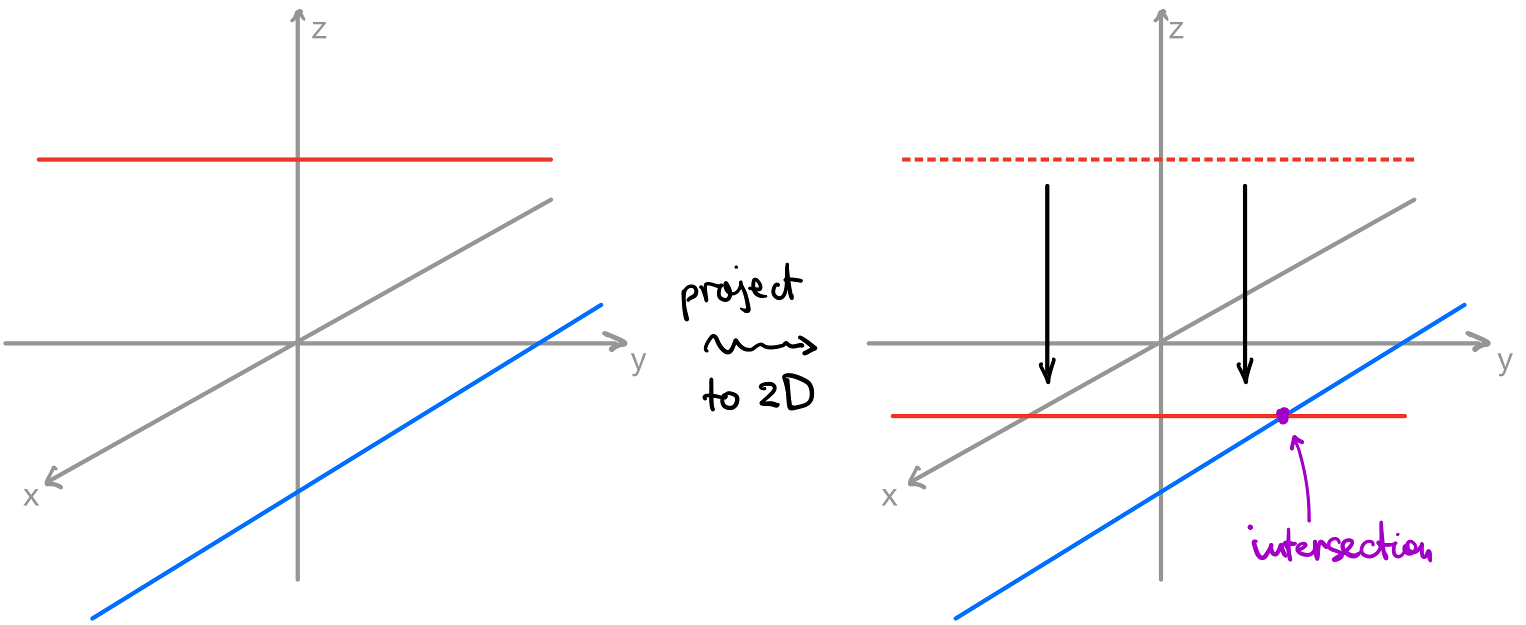

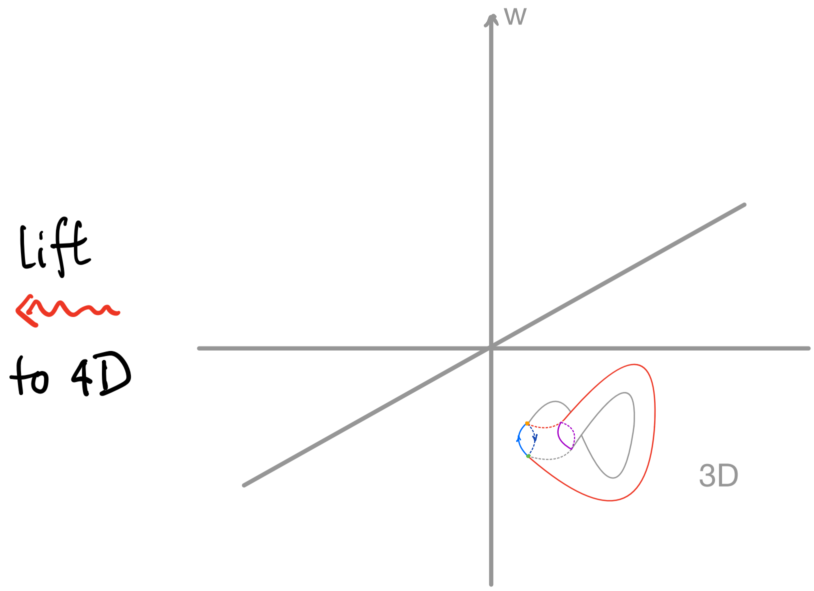

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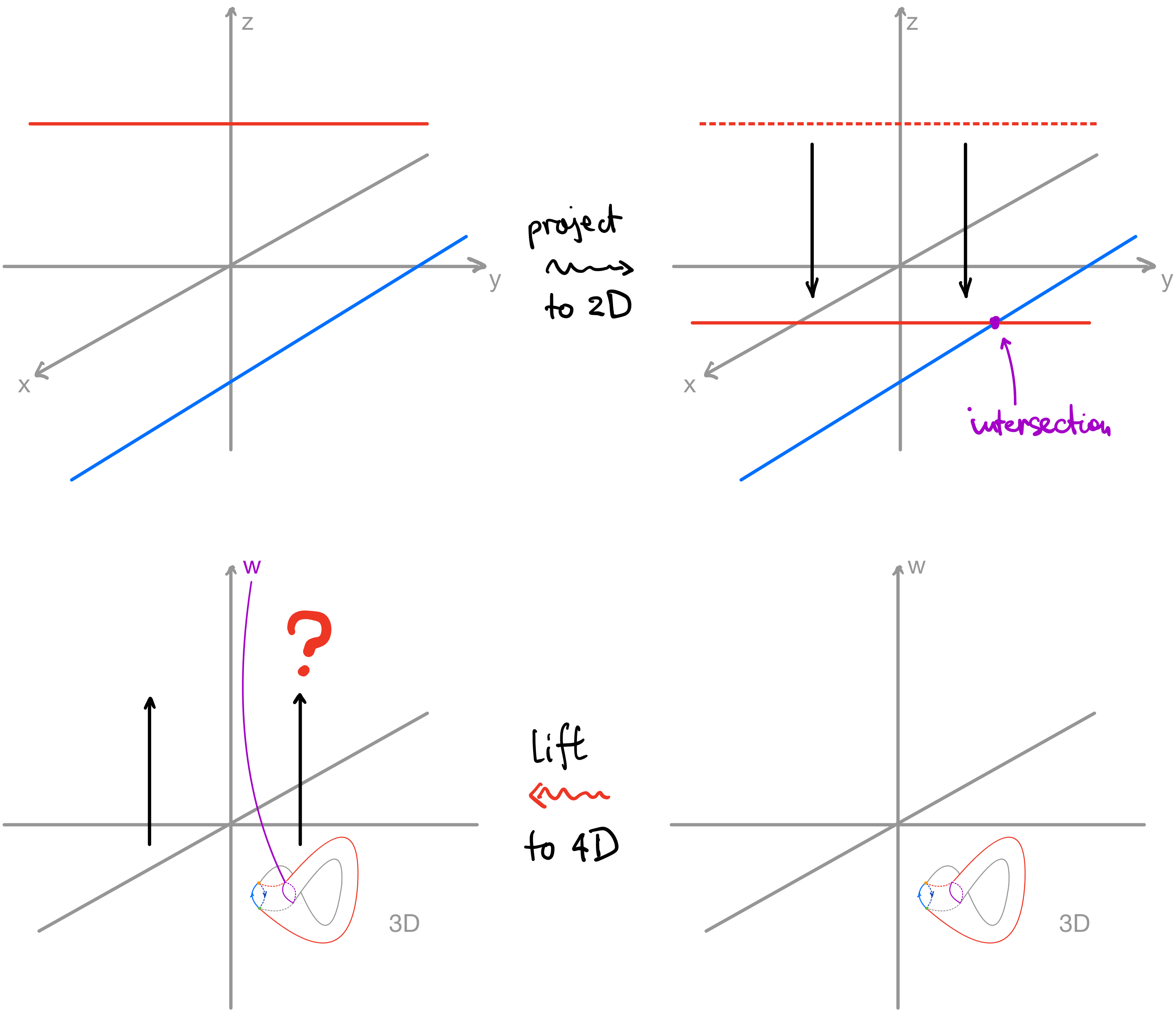

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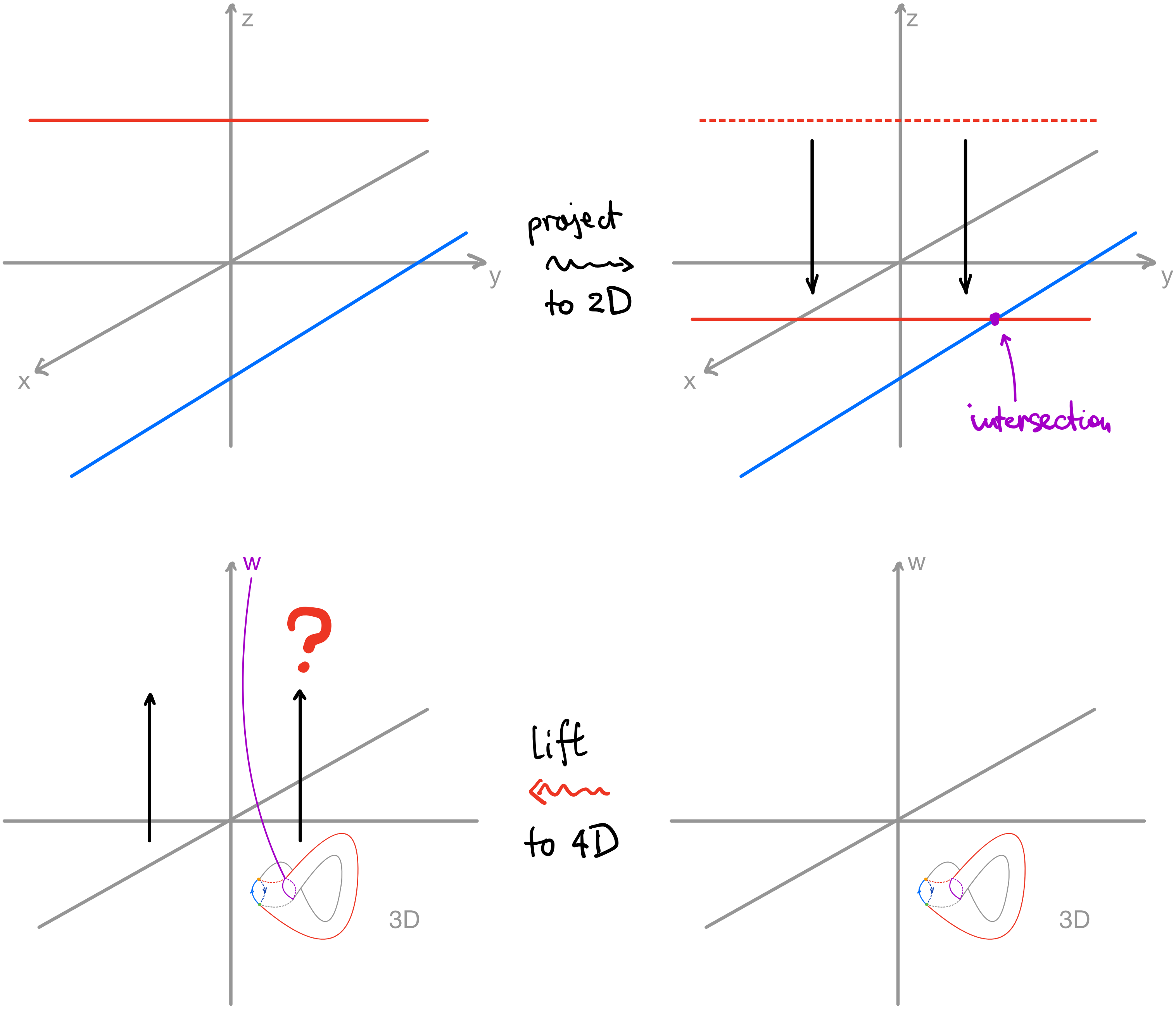

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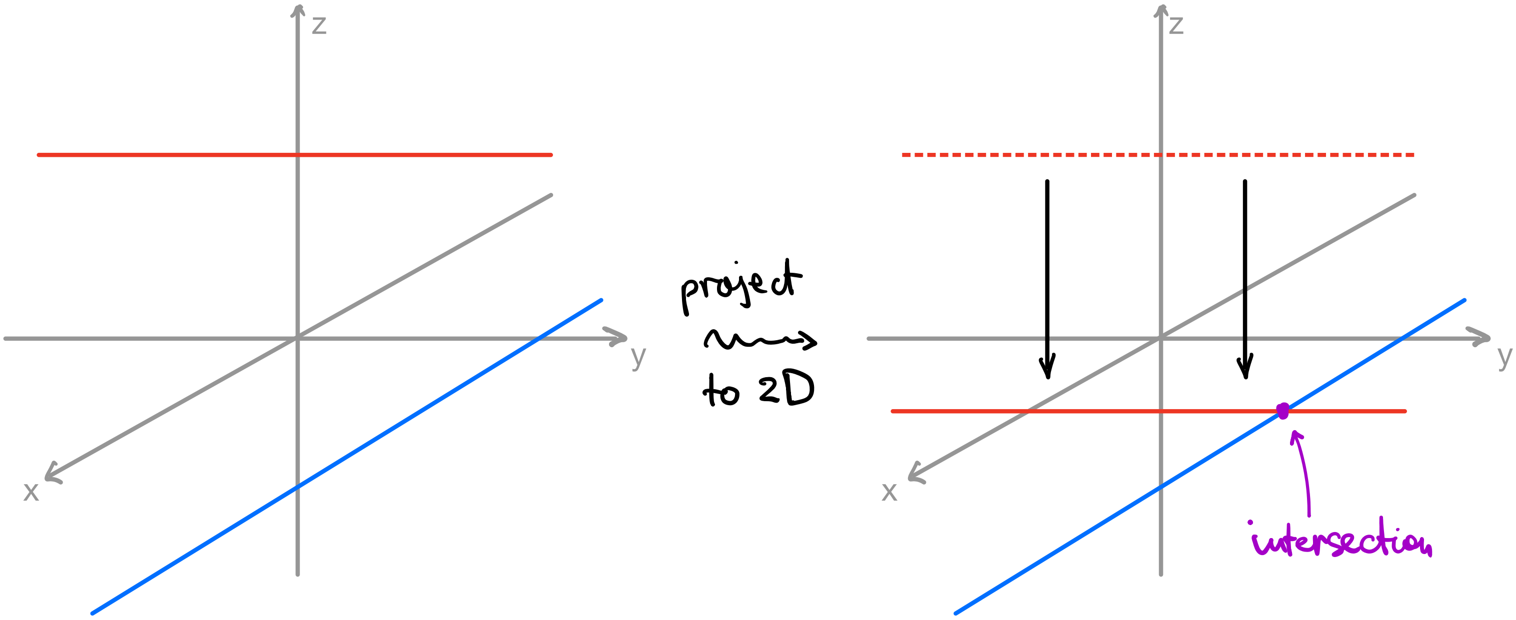

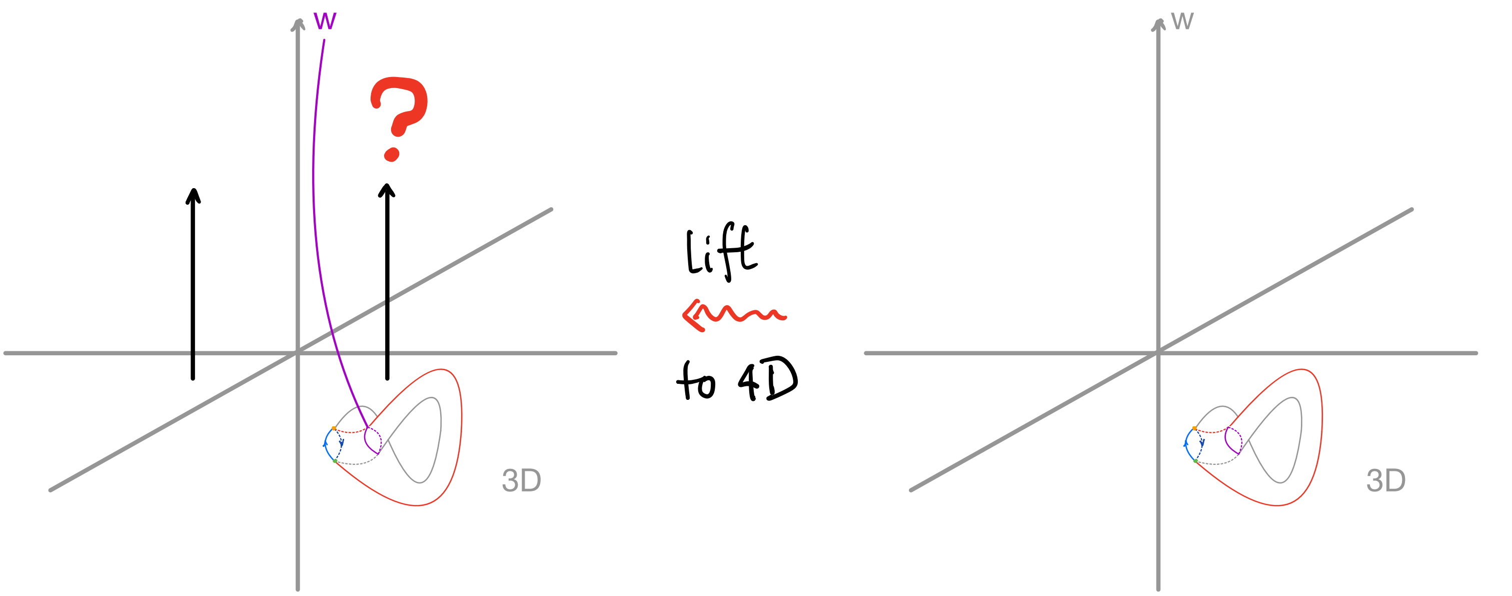

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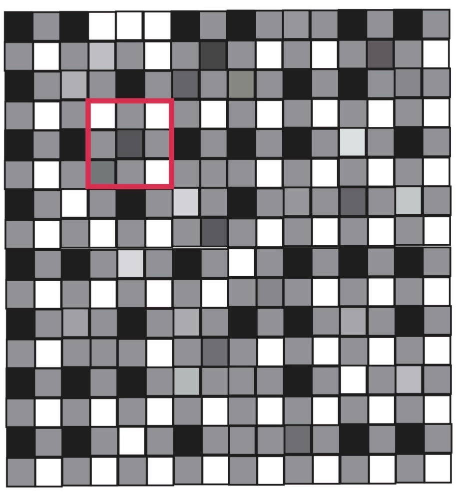

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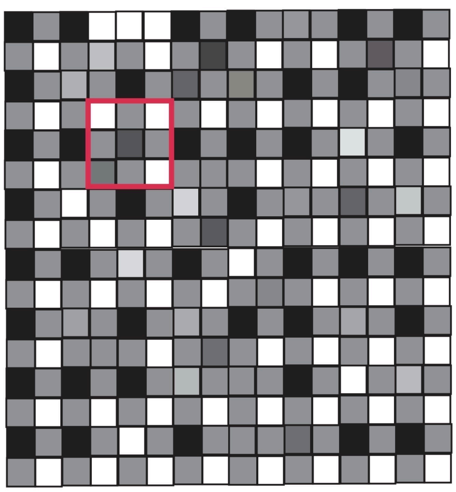

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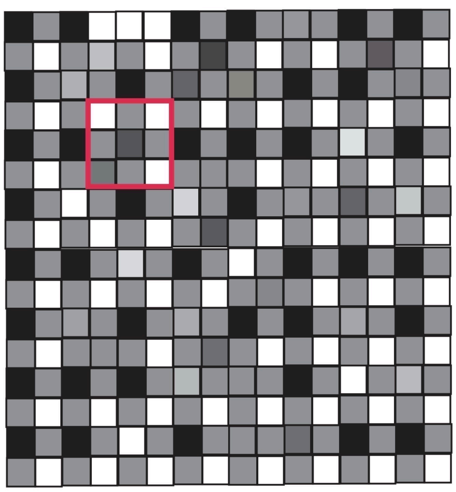

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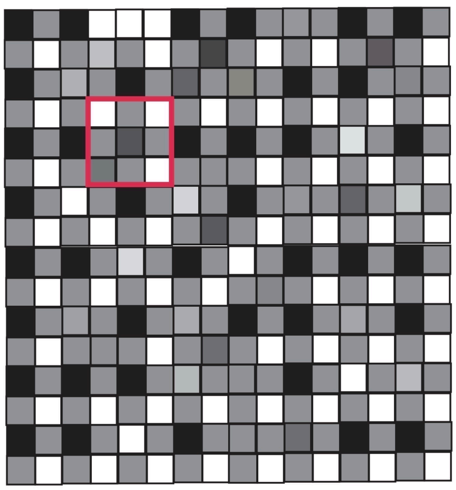

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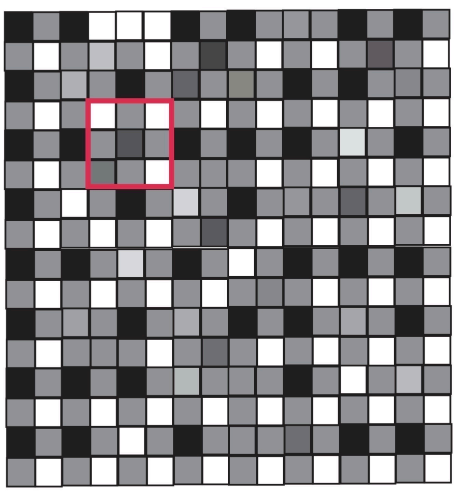

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3. Low contrast will dominate statistics, not interesting. *High* contrast patches delineate profiles

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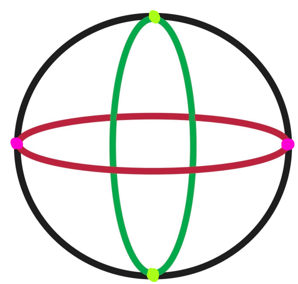

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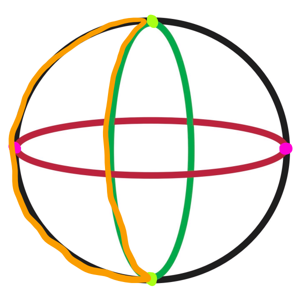

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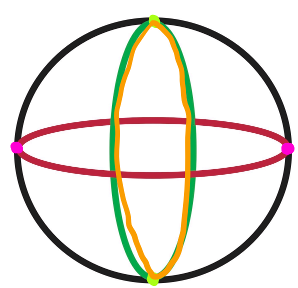

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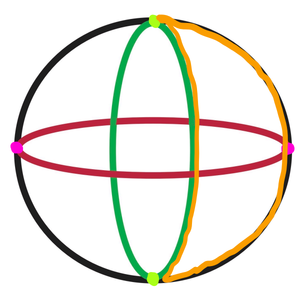

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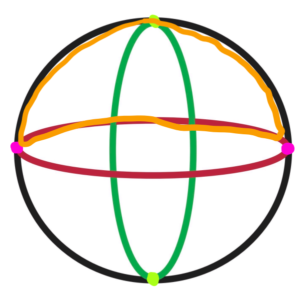

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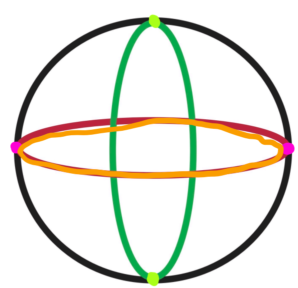

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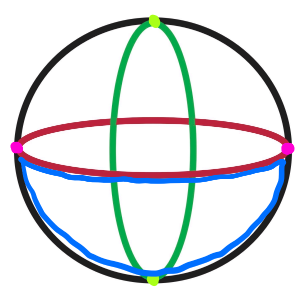

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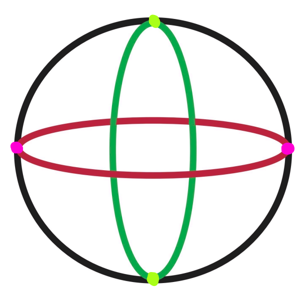

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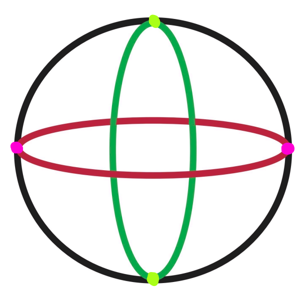

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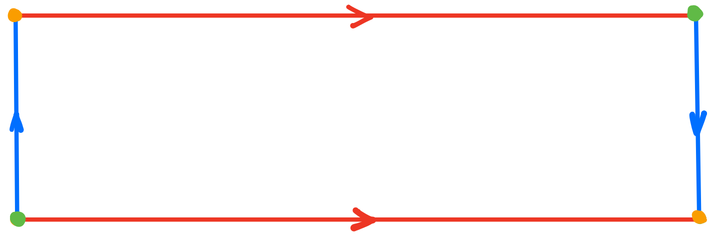

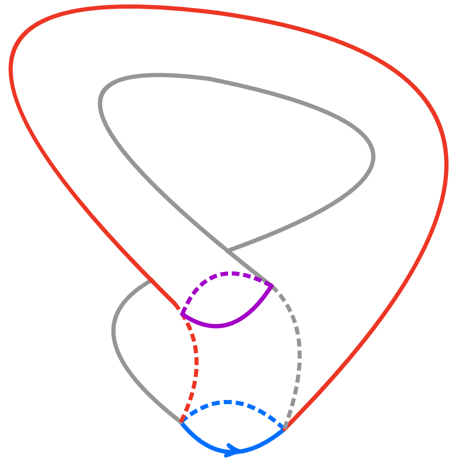

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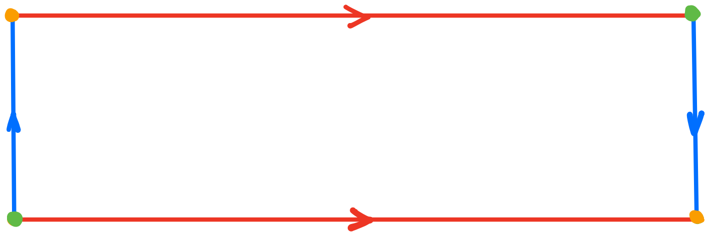

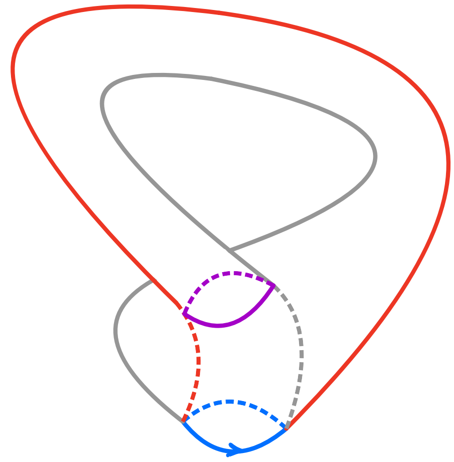

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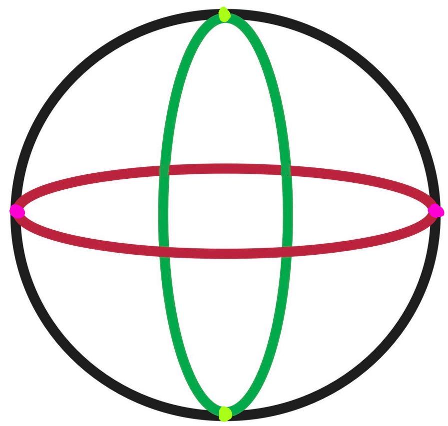

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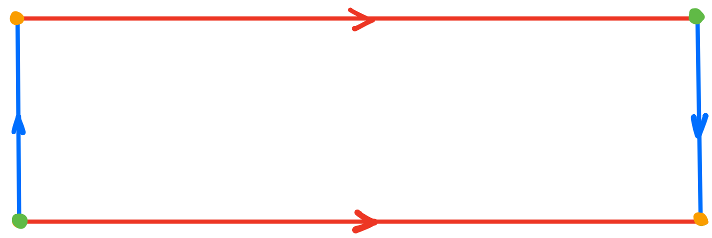

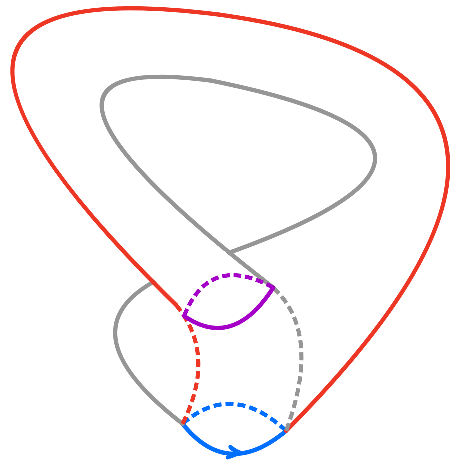

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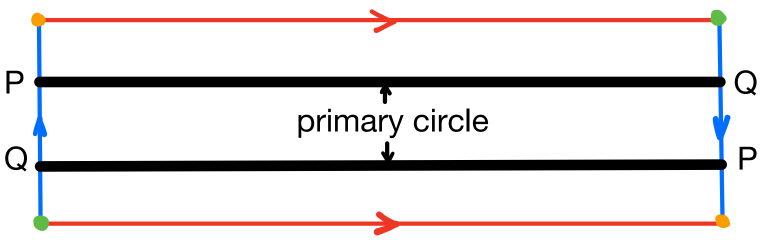

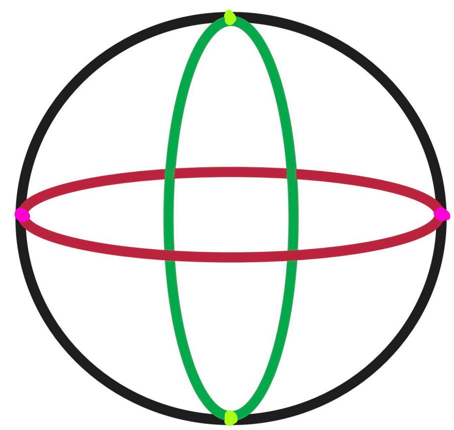

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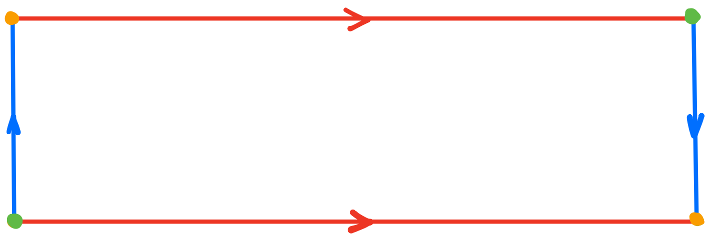

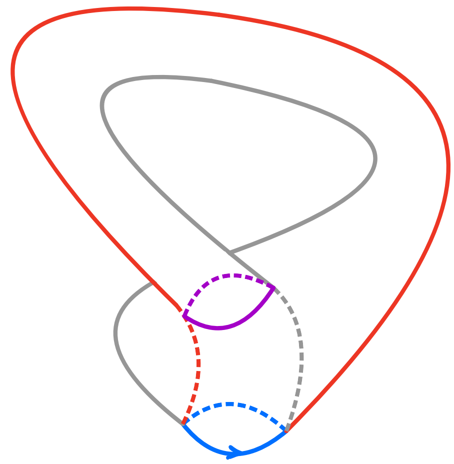

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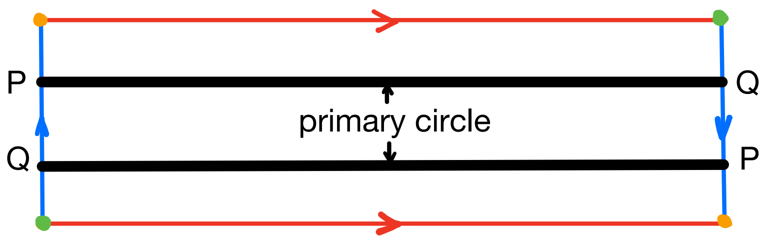

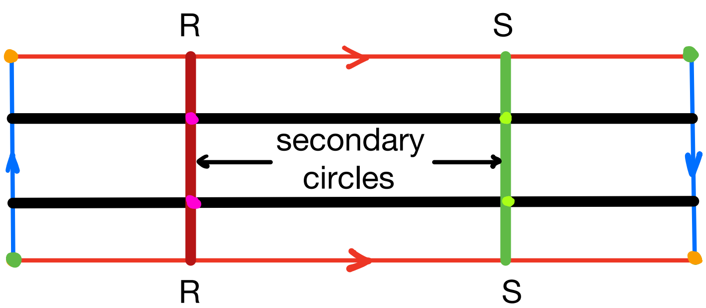

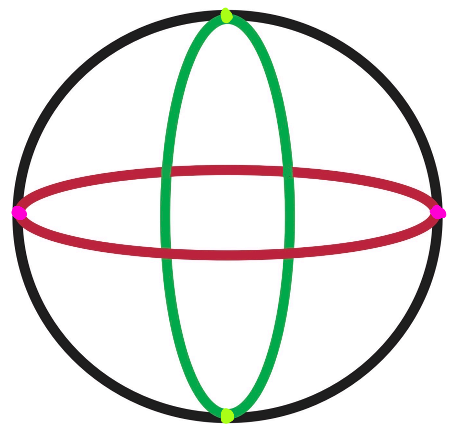

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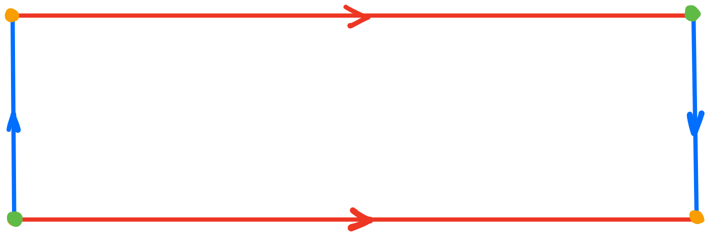

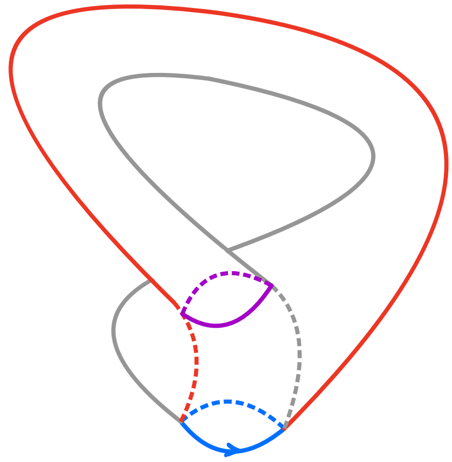

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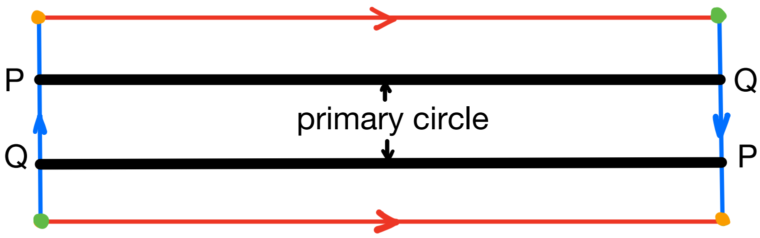

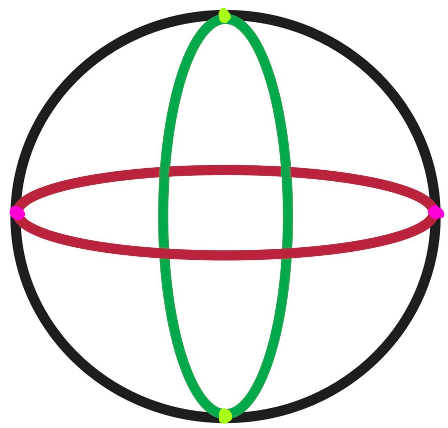

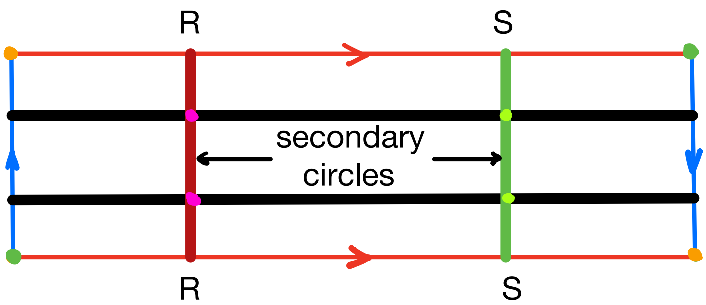

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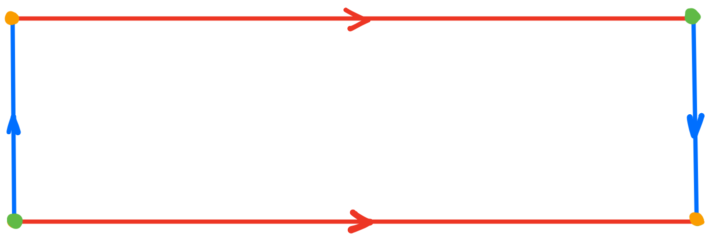

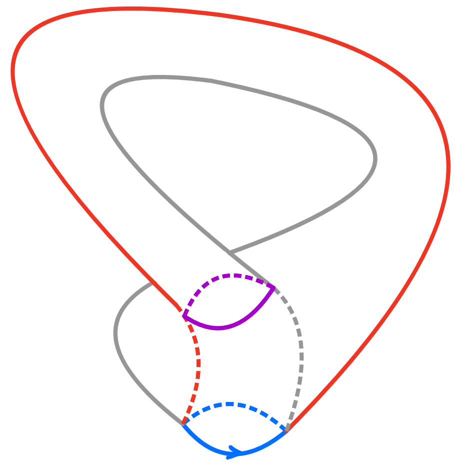

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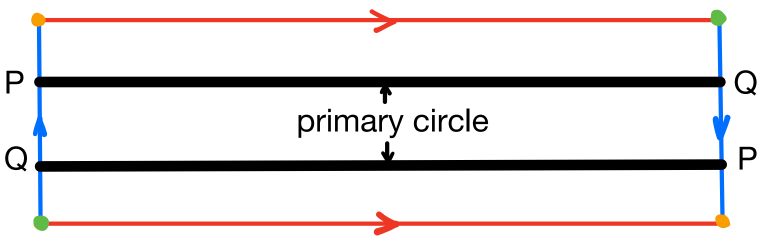

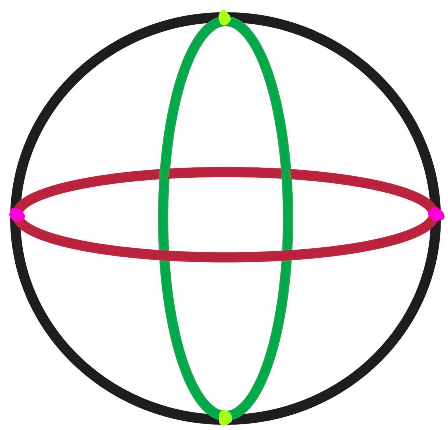

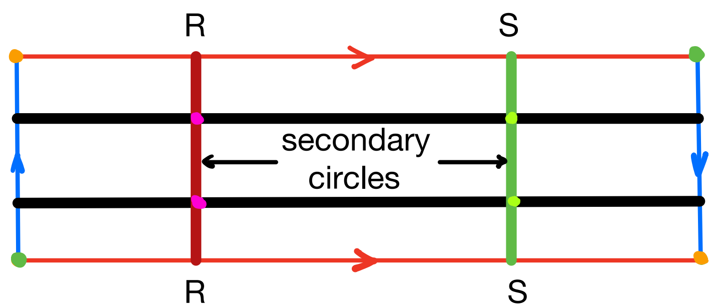

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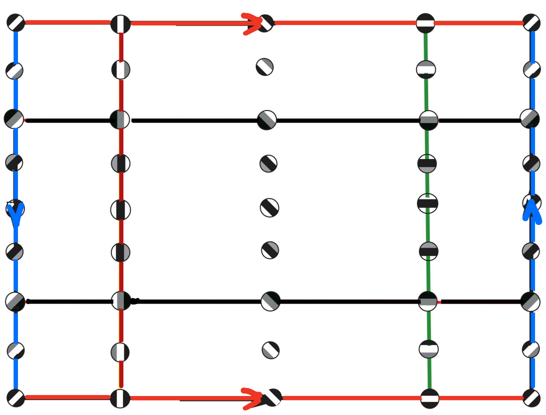

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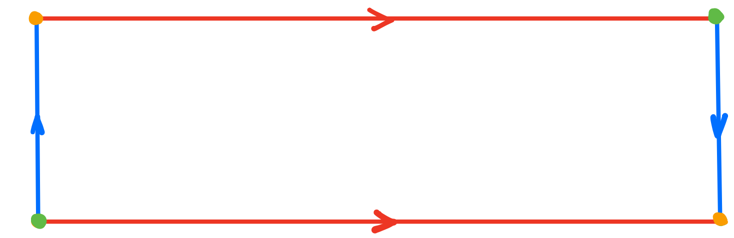

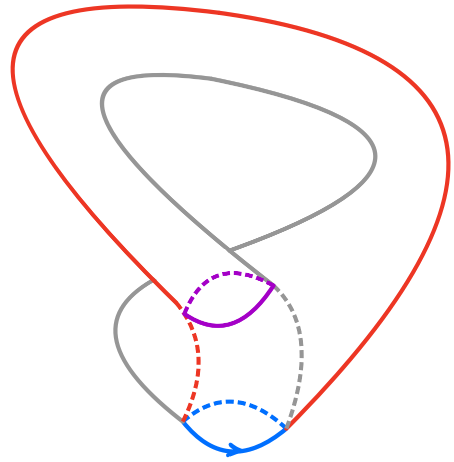

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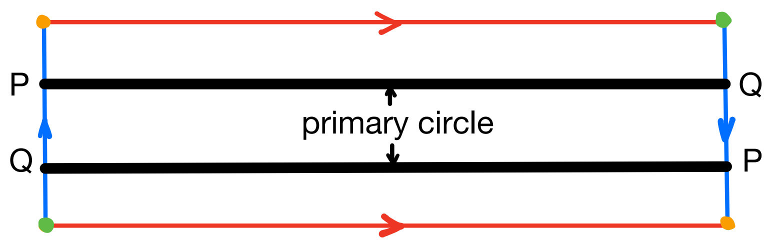

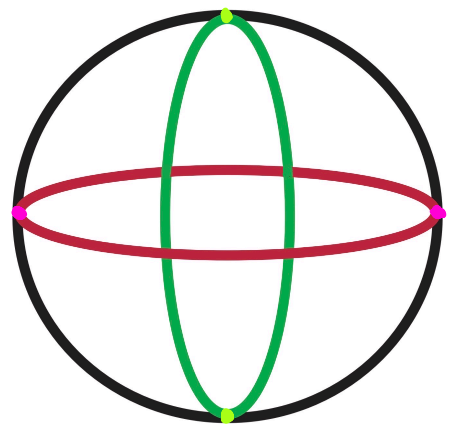

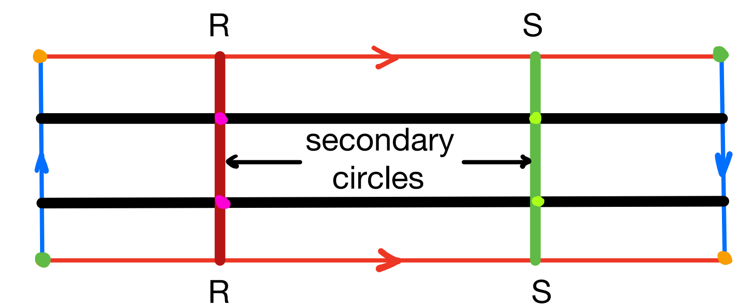

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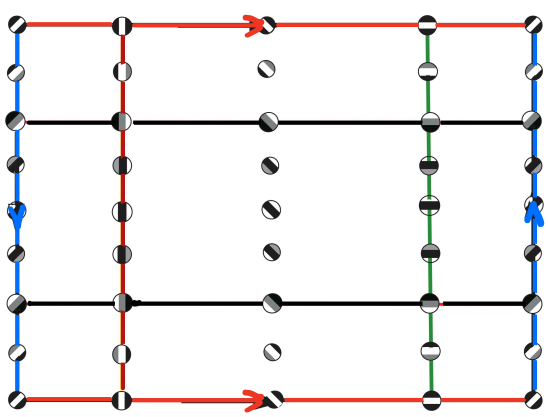

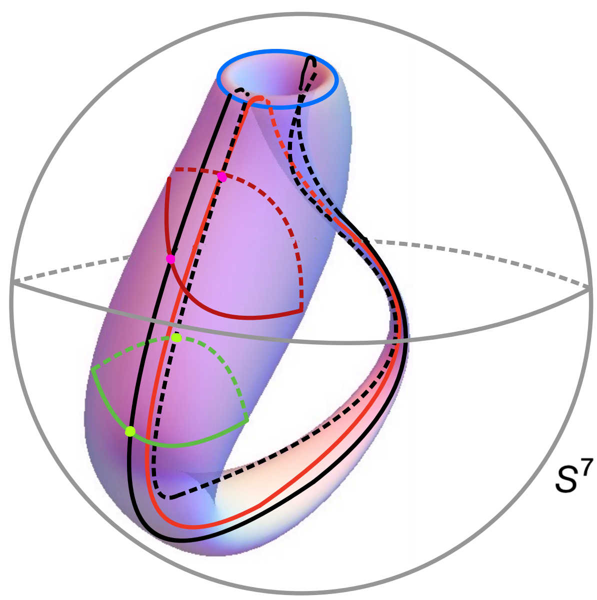

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Other weird surfaces contain **singularity**, such as

Swallowtail

- 2D motion visualization in Mathematica
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Where does it come from mathematically?

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围绕奇异点的拓扑超表面

16:00 – 17:00
2021年10月14日（星期四）
工学院南楼326

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Assistant Professor
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A topological classification for intersection singularities of exceptional surfaces in pseudo-Hermitian systems

Hongwei Jia[#], Ruo-Yang Zhang[#], Jing Hu, Yixin Xiao, Yifei Zhu[*], C. T. Chan[†]

Abstract: Exceptional degeneracy plays a pivotal role in the topology of non-Hermitian systems, and recently many efforts have been devoted to classifying exceptional points and exploring the intriguing physics. However, intersections of exceptional surfaces, which are commonly present in non-Hermitian systems with parity-time symmetry or chiral symmetry, were not classified. Here we classify generic pseudo-Hermitian systems, for which the momentum space is partitioned by exceptional surfaces, and these surfaces intersect stably in momentum space. The topology of such gapless structure can be viewed from its quotient space, which is "figure eight," by considering the equivalence relations of eigenstates in energy gaps and on exceptional surfaces. We reveal that the topology of such systems can be described by a free non-Abelian group composed of products of two generators. The topological invariants in the group are well associated with the spin rotation of eigenstates via adiabatic transformations. Our classification does not rely on specific bandgaps and is thus a global topological description. Importantly, the classification predicts a new phase of matter, and can systematically explain how the exceptional surfaces and their intersections evolve against deformations to the Hamiltonian. Our work opens a new pathway for designing systems with robust topological phases, and is potentially a guidance for applications to sensing and lasing devices utilizing exceptional surfaces and intersections.

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Thank you, and Happy π Day!

Credits and references

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- People walk on the Wuchazi Bridge: China News Service via Getty Images
- Wuchazi Bridge day: https://www.archdaily.com/945541/wuchazi-bridge-scsj/5f326cbab35765e6aa000155-wuchazi-bridge-scsj-photo
- Wuchazi Bridge night: https://www.dezeen.com/2020/08/03/chengdu-infinity-loop-wuchazi-bridge
- Design concept of the Wuchazi Bridge quote: https://www.archdaily.cn/cn/945542/wu-cha-zi-da-qiao-si-chuan-sheng-jian-zhu-she-ji-yan-jiu-yuan
- Wuchazi Bridge design picture: https://www.tomwuenschmann.de/projects/infinity-loop
- Meixi Lake Bridge pictures: NEXT Architects
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- Non-orientable shapes cartoon: https://www.reddit.com/r/surrealmemes/comments/8t0jy7/have_you_eaten_your_s_h_a_p_e_s_today
- Natural image statistics example, including images, adapted from Gunnar Carlsson's slides, *Topology of data*, available at http://www.stat.uchicago.edu/~lekheng/meetings/mmds/slides2008/carlsson.pdf
- Gunnar Carlsson, *Topology and data*, Bull. Amer. Math. Soc. (N.S.) **46** (2009), no. 2, 255–308. MR2476414, https://doi.org/10.1090/S0273-0979-09-01249-X
- Lee, A.B., Pedersen, K.S. & Mumford, D. The Nonlinear Statistics of High-Contrast Patches in Natural Images. *International Journal of Computer Vision* **54**, 83–103 (2003). https://doi.org/10.1023/A:1023705401078
- Carlsson, G., Ishkhanov, T., de Silva, V. et al. On the Local Behavior of Spaces of Natural Images. *International Journal of Computer Vision* **76**, 1–12 (2008). https://doi.org/10.1007/s11263-007-0056-x
- Swallowtail 3D visualization: Mathematica, https://yifeizhu.github.io/swtl.nb
- Swallowtail 3D movie: Oliver Labs, https://yifeizhu.github.io/swtl.mp4
- Beijing Winter Olympics picture: http://en.kremlin.ru/events/president/news/67715
- Qinghua Song lecture poster: Department of Electronic and Electrical Engineering, Southern University of Science and Technology
- Hongwei Jia, Ruo-Yang Zhang, Jing Hu, Yixin Xiao, Yifei Zhu, and C. T. Chan, *A topological classification for intersection singularities of exceptional surfaces in pseudo-Hermitian systems*, preprint, https://yifeizhu.github.io/2-band. pdf (supplementary materials, https://yifeizhu.github.io/2-band-supp.pdf), 2022
- Jing Hu, Ruo-Yang Zhang, Yixiao Wang, Yifei Zhu, Hongwei Jia, and C. T. Chan, *Observation of swallowtail catastrophe singularity in non-Hermitian bands and its topological origin*, preprint, https://yifeizhu.github.io/3-band.pdf (supplementary materials, https://yifeizhu.github.io/3-band-supp.pdf), 2022
- Michael Atiyah quote: http://www.picturequotes.com/michael-atiyah-quotes