# ZHU Yifei Research Group

Department of Mathematics

Southern University of Science and Technology

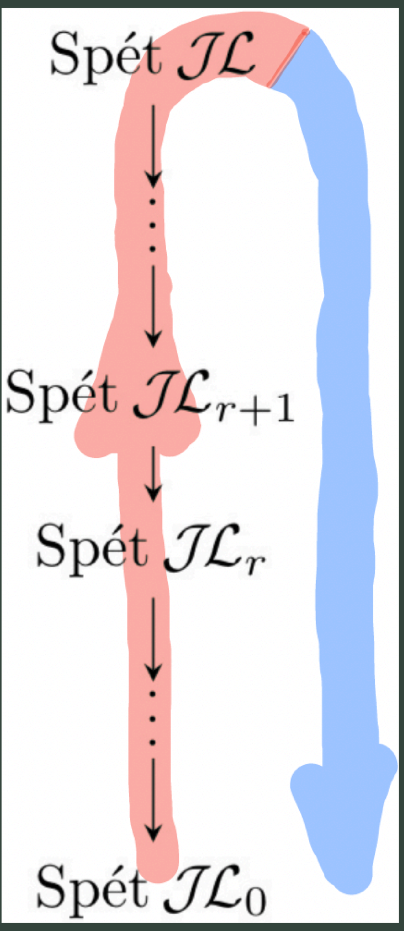
## Moduli, moduli, moduli

Our current research concentrates on interaction of algebraic topology with algebraic geometry and number theory, especially moduli spaces from spectral algebraic geometry in the context of the Langlands program. We also work on application of topology and geometry to interdisciplinary research, including condensed matter physics and materials science, time series analysis, human–computer interaction, and systems science. Here feature three sorts of moduli spaces we've studied.

## Spheres, spectral algebraic geometry, and Jacquet–Langlands correspondence

Given an $\mathbb{E}$-infinity ring spectrum R, with motivation from structure and computation centered around the homotopy groups of spheres, we defined relative effective Cartier divisors for a spectral Deligne–Mumford stack and proved that, as a functor from connective R-algebras to topological spaces, it is relatively representable. We then solved various moduli problems of level structures on spectral abelian varieties, overcoming difficulty at primes dividing the level. In particular, we obtained higher-homotopical refinement for finite levels of the Lubin–Tate tower as $\mathbb{E}$-infinity rings, which generalize Morava, Hopkins, Miller, Goerss, and Lurie's spectral realization at the ground level. Moreover, passing to the infinite level and then descending along the equivariantly isomorphic Drinfeld tower, we obtained a Jacquet–Langlands dual to the Morava E-theory spectrum, along with homotopy fixed point spectral sequences dual to those studied by Devinatz and Hopkins. These serve as potential tools for computing higher-periodic homotopy types from pro-étale cohomology of p-adic general linear groups.

- Xuecai Ma and Yifei Zhu, Spectral moduli problems for level structures and an integral Jacquet–Langlands dual of Morava E-theory, Preprint (2025), 40 pages.

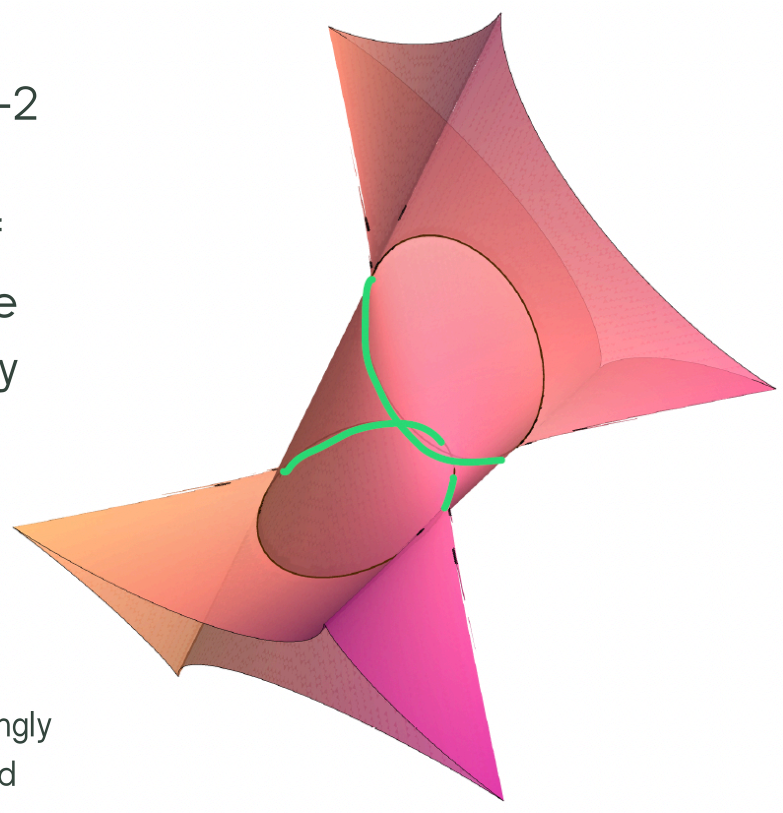



Climbing up to the perfectoid level along the Lubin–Tate tower of moduli for deformations of a formal group, and then climbing down along the Drinfeld tower of moduli for deformations of a shtuka

## Quantum materials, Higgs bundles, and geometric Langlands correspondence

With motivation from condensed matter physics and materials science in collaboration with physicists, we explored the intriguing topological structures arising from certain novel non-Hermitian quantum mechanical systems, whose moduli spaces have stratified, non-isolated singularities, as well as their circuit realizations and extraordinary physical consequences. Mathematically, we investigated the topology and geometry of explicit examples of rank-2 and rank-3 Higgs bundles, as eigenbundles associated to these gapless systems. In particular, we explained why hyperbolic band theory (after A. J. Kollár et al.) and such non-Hermitian systems are natural from the viewpoint of Higgs bundles. Moreover, afforded by these examples, we explained how the physical bulk–edge correspondence may have a mathematical origin of the Langlands correspondence, as evidenced by the respective roles played by Higgs bundles.

- Yifei Zhu, with Jing Hu, Ruo-Yang Zhang, Yixiao Wang, Xiaoping Ouyang, Hongwei Jia, and C. T. Chan, Non-Hermitian swallowtail catastrophe revealing transitions among diverse topological singularities, Nature Physics 19 (2023), 1098–1103.



This swallowtail-quadruple parameter space supports an interestingly ramified Higgs bundle as the eigenbundle of a non-Hermitian 3-band Hamiltonian, where the developable surface of multiple eigenvalues conjecturally encloses a 3D hyperbolic space.

## Human speech signals, persistent homology, and topological distribution spaces of data

In artificial-intelligence-aided signal processing, existing deep learning models often exhibit a black-box structure. We demonstrated that topological methods not only effectively capture intrinsic and complex structural information but can also be integrated into neural networks. We provided a transparent methodology, TopCap, to capture topological features inherent in time series for basic machine learning. Compared to prior approaches, we obtained descriptors which probe finer information such as the vibration of a time series. Notably, in classifying voiced and voiceless consonants, TopCap achieves an accuracy exceeding 96%, consistently standing in comparison with state-of-the-art neural networks. Moreover, we integrated TopCap features into those neural networks, making a network more interpretable with better performance in terms of accuracy, steadiness, convergence of loss function, and robustness against noise. By analogy to image and video data, a main goal remains to use topological methods to reveal distribution spaces for speech/audio data, and further apply these topological inputs for smarter learning.

- Pingyao Feng, Qingrui Qu, Haiyu Zhang, Siheng Yi, Zhiwang Yu, Zeyang Ding, and Yifei Zhu, Topology-enhanced machine learning for consonant recognition, Preprint (2024), 20 pages.

## Thanks

Our research group has been growing with the Department of Mathematics, and we wish her a Happy $10^{\text{th}}$ Birthday!

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