Symmetry encoded by norm maps

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Workshop on loop spaces, supersymmetry and index theory 2017

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$$t + \frac{g(\mathbb{CP}^2)}{3}t^3 + \frac{g(\mathbb{CP}^4)}{5}t^5 + \cdots = \int_0^t \frac{dx}{\sqrt{1 - 2\delta x^2 + \epsilon x^4}}$$

for some constants $\delta, \epsilon \in R$.

Examples signature ($\delta = \epsilon = 1$), $\hat{A}$-genus ($\delta = -1/8, \epsilon = 0$)
The universal elliptic genus, with $R = \mathbb{C}[\delta_2, \epsilon_4] \cong \mathrm{MF}\big(\Gamma_0(2)\big)_\bullet \otimes \mathbb{C}$

$$\begin{tikzcd}
MO\langle 2\rangle^{-*}(\mathrm{pt}) = \Omega_*^{SO} \arrow[r, "w", "\text{Witten '87}"'] & \mathbb{C}[\![q]\!] \\
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MO\langle 8\rangle^{-*}(\mathrm{pt}) = \Omega_*^{String} \arrow[r] \arrow[u, hook] & \mathrm{MF}_\bullet \cong \mathbb{Z}[c_4, c_6, (c_4^3 - c_6^2)/(12)^3] \arrow[u, hook]
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Index theory (Witten '87, '88) Given a spin manifold M, its genus $w(M)$ = the S^1-equivariant index of a Dirac operator on the free loop space $\mathcal{L}M$. supercharge of a supersymmetric nonlinear sigma model

Symmetries (Eguchi-Ooguri-Tachikawa 2011) The elliptic genus of the K3 surface has a natural decomposition in terms of dimensions of irreducible representations of the largest Mathieu group M_{24}. $|M_{24}| = 2^{10} \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 23$

$X^4 + Y^4 + Z^4 + W^4 = 0$ in $\mathbb{CP}^3$ has symmetry $G \subset M_{24}$ with $|G| = 2^7 \cdot 3$

(Gannon '16) proof of the M_{24}-moonshine for this Jacobi form

(Duncan-Griffin-Ono '15) 22 more moonshines for mock modular forms

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topological modular forms

an elliptic cohomology theory

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$D_n(-) := (-)^{\wedge n}/\Sigma_n, \quad n \geq 0$

H_∞ means commutative up to homotopy

The vertical maps correspond to *power operations*; e.g., given

$$E^0 X \cong \pi_0 E^{\Sigma_+^\infty X} =: \pi_0 A = [S, A]_S \cong [E, A]_E \ni f$$

we have

$$E \xrightarrow{\alpha} D_n E \xrightarrow{D_n f} D_n A \to A$$

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$$\begin{tikzcd} D_n MU\langle 6\rangle \arrow[r, "D_n g"] \arrow[d] & D_n E \arrow[d] \\ MU\langle 6\rangle \arrow[r, "g"] & E \end{tikzcd}$$

$D_n(-) \coloneqq (-)^{\wedge n}/\Sigma_n, \quad n \geq 0$

H_∞ means commutative up to homotopy

The vertical maps correspond to *power operations*; e.g., given

$$E^0 X \cong \pi_0 E^{\Sigma^\infty_+ X} =: \pi_0 A = [S, A]_S \cong [E, A]_E \ni f$$

we have

$$E \xrightarrow{\alpha} D_n E \xrightarrow{D_n f} D_n A \to A$$

and thus a power operation $Q_\alpha \colon E^0 X \to E^0 X$.
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Elliptic cohomology and Morava E-theories

Definition (Ando-Hopkins-Strickland '01, Lurie '09)

elliptic cohomology theory $= \left\{ \begin{array}{l} R, \quad C/R, \quad E, \\ E^0(\mathrm{pt}) \cong R,\ \operatorname{Spf} E^0(\mathbb{CP}^\infty) \cong \widehat{C} \end{array} \right\}$

- A *formal group* G is a group object in formal schemes.
- A *formal group law* F is G with a chosen coordinate t:

$$\mathcal{O}_G \cong R[\![t]\!] \ \rightsquigarrow \ R[\![t_1, t_2]\!] \ni F\big(t(P_1), t(P_2)\big) = t(P_1 \underset{G}{+} P_2)$$

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$$\mathcal{E}\colon \{\text{formal groups over perfect fields}\} \to \{E_\infty\text{-ring spectra}\}$$

- $\mathrm{Spf}\, E^0(\mathbb{CP}^\infty)$ = universal deformation of a fg Γ of height n over a perfect field k of char p
- $\pi_* E \cong \mathbb{W}(k)[\![u_1, \ldots, u_{n-1}]\!][u^{\pm 1}], \quad |u| = 2$

A *deformation* (G, i, η) of Γ/k to R (Lubin-Tate '66):

$$\begin{tikzcd}
G \arrow[d] & \pi^*G \arrow[l] \arrow[r, "\eta"] \arrow[d] \arrow[dl, phantom, "\llcorner"] & i^*\Gamma \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner"] & \Gamma \arrow[d] \\
\mathrm{Spf}\, R & \mathrm{Spf}\, R/\mathfrak{m} \arrow[l, "\pi"'] \arrow[r, equal] & \mathrm{Spf}\, R/\mathfrak{m} \arrow[r, "i"] & \mathrm{Spf}\, k
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H$_\infty$ $MU\langle 0\rangle$-orientations for Morava E-theories

Theorem (Z.)

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Remarks

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Let k be an algebraic extension of $\mathbb{F}_p$, Γ be a formal group over k of height n, and E be the Morava E-theory associated to Γ/k. Given any coordinate x_Γ on Γ, there exists a unique coordinate x on G_E lifting x_Γ such that $MU\langle 0\rangle \xrightarrow{x} E$ is an H_∞ map.

Remarks

- (Ando '95) $k = \mathbb{F}_p$, $\Gamma =$ Honda fg, $E \neq$ elliptic cohomology
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Fix Γ/k. Then deformations of Frobenius $(G, i, \eta) \to (G', i', \eta')$ are classified by rings A_r, $r \geq 0$, with p^r the order of the subgroup scheme $\ker(G \to G') \subset G$.

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A criterion for H$_\infty$ ortns (Ando '95, Ando-Hopkins-Strickland '04)
Let E be the Morava E-theory associated to Γ/k as before. Then the orientation $MU\langle 0\rangle \to E$ is an H$_\infty$ map if and only if its corresponding coordinate on G_E is norm-coherent.

Theorem (Z.)

- Any coordinate on Γ over k extends uniquely to a norm-coherent coordinate on G_E over $\pi_0 E$.
- This construction is functorial under base change of Γ/k, under k-isogeny out of Γ, and under k-Galois descent.

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