Symmetry encoded by norm maps

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Southern University of Science and Technology

Workshop on loop spaces, supersymmetry and index theory 2017

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for some constants $\delta, \epsilon \in R$.

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Index theory (Witten '87, '88) Given a spin manifold M, its genus $w(M) = \text{the } S^1$ -equivariant index of a Dirac operator on the free loop space $\mathcal{L}M$. supercharge of a supersymmetric nonlinear sigma model

Symmetries (Eguchi-Ooguri-Tachikawa 2011) The elliptic genus of the K3 surface has a natural decomposition in terms of dimensions of irreducible representations of the largest Mathieu group M_{24} . $|M_{24}| = 2^{10} \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 23$

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(Gannon '16) proof of the M_{24} -moonshine for this Jacobi form

(Duncan-Griffin-Ono '15) 22 more moonshines for mock modular forms



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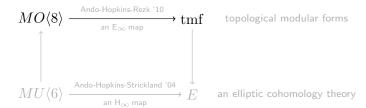
(Atiyah-Bott-Shapiro '64) \hat{A} -orientation: $MO\langle 4 \rangle \to KO$

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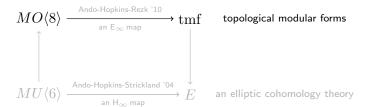
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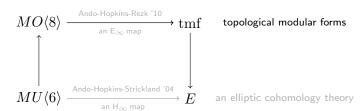
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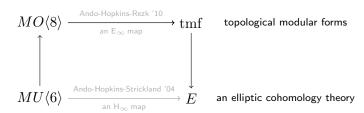
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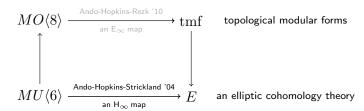


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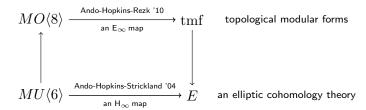
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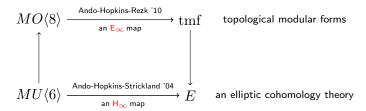
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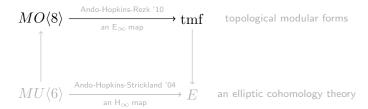


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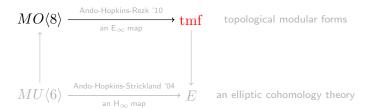
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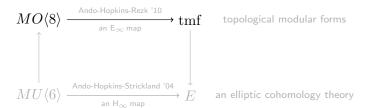


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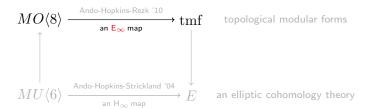
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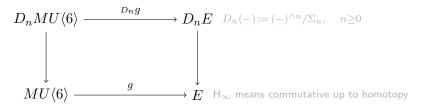


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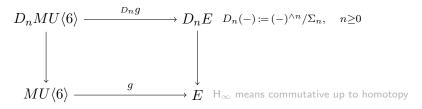
$$E^{0}X \cong \pi_{0}E^{\sum_{+}^{\infty}X} =: \pi_{0}A = [S, A]_{S} \cong [E, A]_{E} \ni f$$

we have

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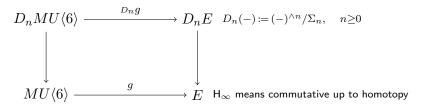
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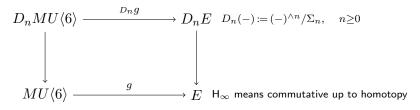
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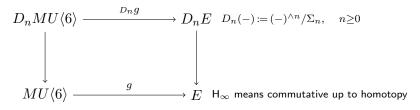
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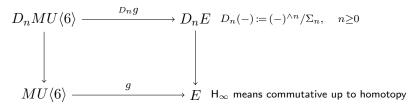
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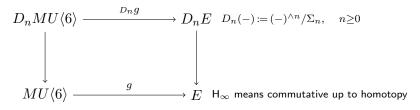
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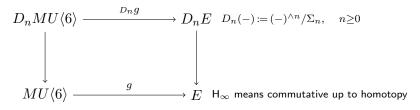
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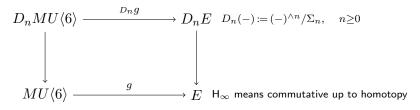
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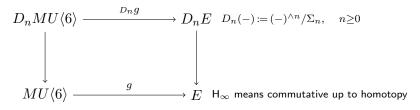
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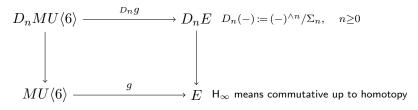
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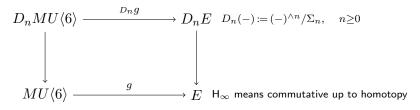
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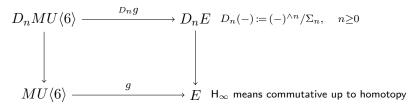
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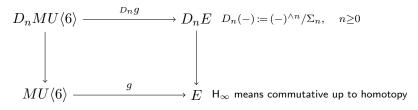
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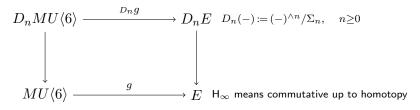
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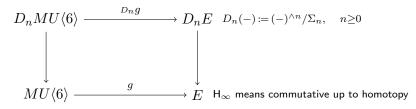
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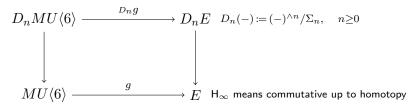
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$$\mathcal{O}_G \cong R[\![t]\!] \rightsquigarrow R[\![t_1, t_2]\!] \ni F(t(P_1), t(P_2)) = t(P_1 + P_2)$$
$$F(c_1(\mathcal{L}_{\text{univ}}), c_1(\mathcal{L}_{\text{univ}})) = c_1(\mathcal{L}_{\text{univ}} \otimes \mathcal{L}_{\text{univ}})$$

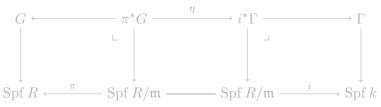
• Coordinates on $G_E := \operatorname{Spf} E^0(\mathbb{CP}^\infty) \leftrightarrow \operatorname{ortns} MU\langle 0 \rangle \to E.$ Question Which coordinates correspond to H_∞ orientations?



Theorem (Morava '78, Goerss-Hopkins-Miller '90s–2004)

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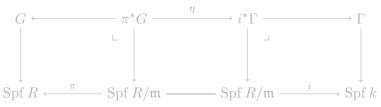
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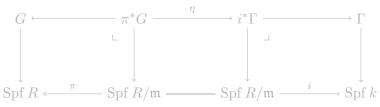
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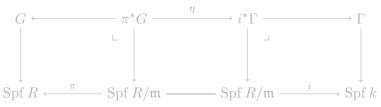
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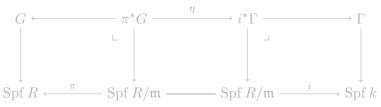
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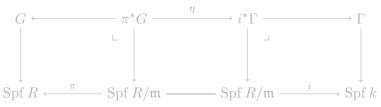
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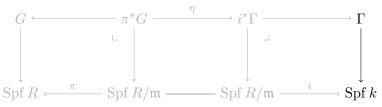
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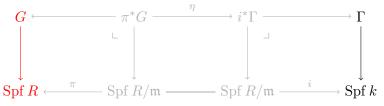
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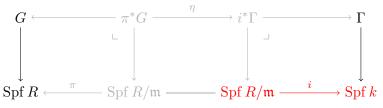
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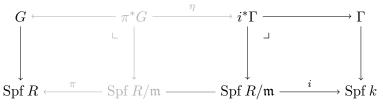
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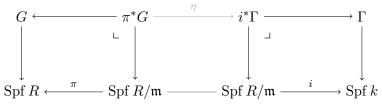
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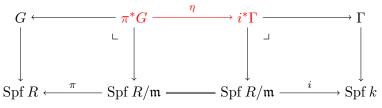
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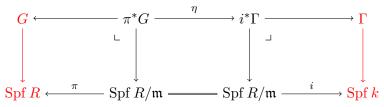
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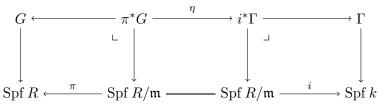
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Let k be an algebraic extension of \mathbb{F}_p , Γ be a formal group over k of height n, and E be the Morava E-theory associated to Γ/k . Given any coordinate x_Γ on Γ , there exists a unique coordinate x on G_E lifting x_Γ such that $MU\langle 0 \rangle \xrightarrow{x} E$ is an H_∞ map.

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Fix Γ/k . Then deformations of Frobenius $(G, i, \eta) \to (G', i', \eta')$ are classified by rings A_r , r > 0, with p^r the order of the subgroup scheme $\ker(G \to G') \subset G$.

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$$\begin{split} H = \text{finite subgroup of } G & f_H \colon G \to G/H & x = \text{coord on } G \\ & \Longrightarrow x_H \coloneqq \operatorname{Norm}_{f_H^*}(x) = \det(\cdot x) \text{ is a coord on } G/H \\ & \mathcal{O}_{G/H} \xrightarrow{f_H^*} \mathcal{O}_G \xrightarrow{\operatorname{Norm}_{f_H^*}} \mathcal{O}_{G/H} \end{split}$$

Explicitly,

$$f_H^*(x_H) = \prod_{Q \in H} \left(x + x(Q) \right)$$

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A coordinate x on G is norm-coherent if

$$\psi_{H}(x) = f_{H}^{*}(x_{H})$$
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A criterion for H_{∞} ortns (Ando '95, Ando-Hopkins-Strickland '04)

Let E be the Morava E-theory associated to Γ/k as before. Then the orientation $MU\langle 0\rangle \to E$ is an H_∞ map if and only if its corresponding coordinate on G_E is norm-coherent.

Theorem (Z.)

- Any coordinate on Γ over k extends uniquely to a norm-coherent coordinate on G_E over $\pi_0 E$.
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