Power operations in elliptic cohomology and related arithmetic topics

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Special session on algebraic and geometric topology 2018

A connection between Topology and Arithmetic (Quillen '69)

stable homotopy theory $\leftrightsquigarrow$ 1-dim formal group laws

complex-oriented $h^*(-)$ $F(x,y)$ over $h^*(\mathrm{pt})$

$$c_1(L_1 \otimes L_2) = F\big(c_1(L_1), c_1(L_2)\big)$$

Example

$$H^*(-;\mathbb{Z}) \leftrightsquigarrow \mathbb{G}_a(x,y) = x + y$$

$$K^*(-) \leftrightsquigarrow \mathbb{G}_m(x,y) = x + y - xy = 1 - (1-x)(1-y)$$

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Background: chromatic homotopy theory 色展同伦论

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elliptic cohomology theory $= \left\{ \begin{array}{ll} E, & C_{E^0(\mathrm{pt})}, \\ \alpha\colon & \operatorname{Spf} E^0(\mathbb{CP}^\infty) \xrightarrow{\sim} \widehat{C} \end{array} \right\}$

Theorem (Morava '78, Goerss-Hopkins-Miller '90s–'04)

$\mathcal{E}\colon$ {formal groups over perfect fields, isos} $\to$ {$\mathbb{E}_\infty$-ring spectra}

- $\operatorname{Spf} E^0(\mathbb{CP}^\infty) =$ the univ deformation of a fg F of height n over a perfect field k of char p
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Elliptic cohomology and Morava E-theory

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Power operations for Morava E-theory

M = E-module $\qquad \pi_0 M = [S, M]_S \cong [E, M]_E$

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$M = E$-module $\qquad \pi_0 M = [S, M]_S \cong [E, M]_E$

$$\mathbb{P}_E(M) = \bigvee_{i \geq 0} \mathbb{P}_E^i(M) = \bigvee_{i \geq 0} (\underbrace{M \wedge_E \cdots \wedge_E M}_{i\text{-fold}})_{h\Sigma_i}$$

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Power operations for Morava E-theory

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Power operations for Morava E-theory (height n prime p)

Theorem (Rezk '09, Barthel-Frankland '13)

If $A = K(n)$-local commutative E-algebra, then

$$\pi_* A = \text{graded amplified } L\text{-complete } \Gamma\text{-ring}$$

- Γ = twisted bialgebra over E_0 (Dyer-Lashof algebra)
- $\exists\, Q_0 \in \Gamma$ with $Q_0(x) \equiv x^p \bmod p$ (Frobenius congruence)

Goal make this structure explicit just as for Dyer-Lashof/Steenrod operations in ordinary homology.

The case of $n = 2$ has been worked out. $\leftsquigarrow$ Arithmetic

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Theorem (Z. '15)

Given any Morava E-theory E of height 2 at a prime p, there is an explicit presentation for its algebra of power operations, in terms of generators $Q_i \colon E^0(-) \to E^0(-)$, $0 \leq i \leq p$, and quadratic relations

$$Q_i Q_0 = -\sum_{k=1}^{p-i} w_0^k \, Q_{i+k} Q_k \; - \sum_{k=1}^{p} \sum_{m=0}^{k-1} w_0^m \, d_{i,\,k-m} \, Q_m Q_k$$

for $1 \leq i \leq p$, where the coefficients w_0 and $d_{i,\,k-m}$ arise from certain modular equations for elliptic curves.

Remark The first example, for $p = 2$, was calculated by Rezk '08. These have been applied to computations in unstable v_2-periodic homotopy theory (Z. '18).

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Moduli of formal groups and algebras of power operations

Recall E-theory at height n and prime p has an underlying model

$$F_k \xleftarrow{\text{univ defo}} \Gamma_{\mathbb{W}(k)[\![u_1,\ldots,u_{n-1}]\!]} \quad \rightsquigarrow \quad E$$

$\circlearrowleft$ Frobenius isogenies — $\circlearrowleft$ power operations

An equivalence of cats (Ando-Hopkins-Strickland '04, Rezk '09)

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Goal Compute one side explicitly to get the other side.

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Moduli of formal groups and moduli of ell. curves (Serre-Tate '64)

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$[\Gamma_0(p)]$ as an *open arithmetic surface* (Katz-Mazur '85)
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Theorem (Z. '15)

A choice of such parameters, h and α, satisfies the equation
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Question At height $n>2$, can we get an explicit presentation for the Dyer-Lashof algebra of Morava E-theory?
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$[\Gamma_0(p)]$ as an *open arithmetic surface* (Katz-Mazur '85)
parameters for its local ring at a supersingular point

Theorem (Z. '15)

A choice of such parameters, h and α, satisfies the equation
$(\alpha - 1)^p(\alpha - p) - \big((-1)^p + (-1)^{p-1}(-p)p + h\big)\alpha = 0$

Question At height $n > 2$, can we get an explicit presentation for the Dyer-Lashof algebra of Morava E-theory?
Investigating J. Weinstein's approach to integral models for modular curves via the *infinite* Lubin-Tate tower.

Moduli of elliptic curves and D.-L. algebras at height 2

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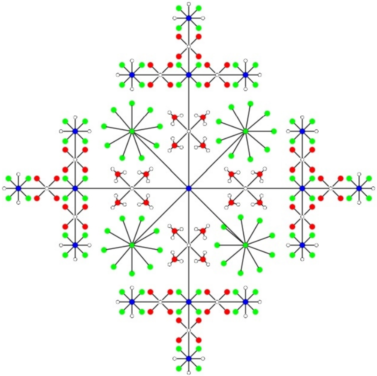
A picture from Jared Weinstein, Semistable models for modular curves of arbitrary level



Thank you.