

MAT8021, Algebraic Topology

Assignment 5

Due in-class on Friday, April 18

1. Suppose X is a CW complex with finitely many cells. Define the Euler characteristic $\chi(X)$ to be the number of even-dimensional cells minus the number of odd-dimensional cells.

If F is *any* field, show that

$$\chi(X) = \sum_i (-1)^i \dim_F(H_i(X; F))$$

(Hint: Express this in terms of the ranks of the boundary maps.)

2. If X is a space with a chosen basepoint, prove $\tilde{H}_{k+1}(S^1 \wedge X) \cong \tilde{H}_k(X)$ for all $k \geq 0$.
3. Compute the following groups.
 - $\text{Tor}_1^{\mathbb{Z}}(\mathbb{Z}/n, \mathbb{Z}/m)$ for $n, m > 0$.
 - $\text{Tor}_k^{\mathbb{Z}/p^2}(\mathbb{Z}/p, \mathbb{Z}/p)$ for $k \geq 0$.
4. Compute all groups $H_k(\mathbb{R}P^2; A)$ for any coefficient group A .