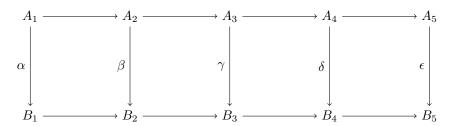
## MAT8021, Algebraic Topology

## Assignment 4

## Due in-class on Friday, March 28

1. (The Five Lemma) Suppose



where the rows are exact and the squares commute. Suppose  $\alpha$ ,  $\beta$ ,  $\delta$ ,  $\epsilon$  are isomorphisms. Show that  $\gamma$  is an isomorphism.

- 2. Prove a stronger version of the Five Lemma: If  $\beta$  and  $\delta$  in the above diagram are injective, and  $\alpha$  is surjective, show that  $\gamma$  is injective.
- 3. Continuing with the previous question, give the dual statement (whose proof is of course essentially the same).
- 4. (Formal Mayer–Vietoris sequence) Suppose that there is a map of long exact sequence as follows:

$$\cdots \longrightarrow F_{n+1} \longrightarrow A_n \longrightarrow B_n \longrightarrow F_n \longrightarrow A_{n-1} \longrightarrow \cdots$$

$$\sim \downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow$$

$$\cdots \longrightarrow G_{n+1} \longrightarrow C_n \longrightarrow D_n \longrightarrow G_n \longrightarrow C_{n-1} \longrightarrow \cdots$$

Here all the maps  $F_n \to G_n$  are isomorphisms. Show that there is a long exact sequence

$$\cdots \to D_{n+1} \to A_n \to B_n \oplus C_n \to D_n \to A_{n-1} \to \cdots$$

(Define the maps first.)