MAT8201, Algebraic Topology

Assignment 1

Due in-class on Friday, February 28

Numbered exercises are from Hatcher's "Algebraic Topology."

1. Suppose X is a CW complex and $\{x_n\}$ is a sequence of points of X such that no two points lie in the interior of the same cell. Show that the sequence $\{x_n\}$ does not converge. Then use this to show that this sequence cannot be contained in any compact set.

2. Prove that a CW complex is locally path connected.

3. Hatcher, Exercise 10 on page 19.

4. Hatcher, Exercise 16 on page 19.

5. Suppose we have a Δ-set X with

$$\begin{aligned} X_0 &= \{p\} \\ X_1 &= \{a, b, c\} \\ X_2 &= \{u, v\} \end{aligned}$$

and face maps

$$\begin{array}{lll} \partial^i(a) = p & \partial^i(b) = p & \partial^i(c) = p \\ \partial^0(u) = a & \partial^1(u) = c & \partial^2(u) = b \\ \partial^0(v) = b & \partial^1(v) = c & \partial^2(v) = a \end{array}$$

What is the resulting space?

6. Construct a Δ-set whose geometric realization is the 2-sphere S^2.