

MAT8021, Algebraic Topology

Assignment 7

Due in-class on Tuesday, April 9

1. Find a pair of chain complexes C_* and D_* such that the tensor product chain complex $C_* \otimes D_*$ does not satisfy the Künneth formula, i.e., there is some n such that

$$H_n(C_* \otimes D_*) \not\cong \bigoplus_{p+q=n} H_p(C_*) \otimes H_q(D_*) \oplus \bigoplus_{p+q=n-1} \text{Tor}(H_p(C_*), H_q(D_*))$$

2. Suppose G is a topological group and X is a topological space with a continuous map $G \times X \rightarrow X$ which is an action of G . Show that $H_*(X)$ becomes a left module over the Pontryagin ring $H_*(G)$.
3. Find the homology of the complex Grassmannian $\text{Gr}_{\mathbb{C}}(3, 5)$.
4. There is a continuous map from one Grassmannian $\text{Gr}(k, n)$ to the next $\text{Gr}(k, n+1)$ by sending a plane $V \subset \mathbb{R}^n$ to the plane

$$\{(0, x_1, \dots, x_n) \mid (x_1, \dots, x_n) \in V\}$$

Show that the image consists of a union of Schubert cells, and find the dimension of the smallest cell not in the image.