

# MAT8021, Algebraic Topology

## Assignment 6

Due in-class on Tuesday, April 2

1. Find all  $(2, 3)$ -shuffles  $\alpha$  and give formulas for the associated shuffle maps  $f_\alpha: \Delta[5] \rightarrow \Delta[2] \times \Delta[3]$ .
2. Find recursive formulas for  $\dim_{\mathbb{Z}/2} H_k((\mathbb{RP}^2)^n; \mathbb{Z}/2)$  in terms of  $k$  and  $n$ .
3. Suppose  $G$  is a topological group and  $x \in H_k(G; \mathbb{Z})$  where  $k$  is odd. Show that  $x^2 = 0$  in  $H_{2k}(G; \mathbb{Z})$ .
4. We saw in class that  $SO(3)$ , the group of  $3 \times 3$  matrices with determinant 1, is a topological group homeomorphic to  $\mathbb{RP}^3$  and so its homology has a Pontryagin ring structure. Show that the square of the generator of  $H_1(SO(3); \mathbb{Z}/2)$  is zero. (Hint: Compare with the Pontryagin ring in homology over  $\mathbb{Z}$ .)