MAT8021, Algebraic Topology

Assignment 5

Due in-class on Tuesday, March 26

1. If X is a space with a chosen basepoint, prove $\tilde{H}_{k+1}(S^1 \wedge X) \cong \tilde{H}_k(X)$ for all $k \geq 0$.

2. Compute the following groups.
   - $\mathrm{Tor}_1^{\mathbb{Z}}(\mathbb{Z}/n, \mathbb{Z}/m)$ for $n, m > 0$.
   - $\mathrm{Tor}_k^{\mathbb{Z}/p^2}(\mathbb{Z}/p, \mathbb{Z}/p)$ for $k \geq 0$.

3. Compute all groups $H_k(\mathbb{RP}^2; A)$ for any coefficient group A.

4. A *differential graded algebra* is a chain complex A_* with associative multiplication maps $\cdot\colon A_p \times A_q \to A_{p+q}$ satisfying the Leibniz rule

   $$\partial(x \cdot y) = (\partial x) \cdot y + (-1)^p x \cdot (\partial y)$$

   for $x \in A_p, y \in A_q$.

   Show that given elements $[x] \in H_p(A)$ and $[y] \in H_q(A)$, we get a well-defined element $[x] \cdot [y]$ in $H_{p+q}(A)$. Show that this makes $H_*(A)$ into a graded ring.