MAT8021, Algebraic Topology

Assignment 3

Due in-class on Tuesday, March 12

Numbered exercises are from Hatcher's "Algebraic Topology."

1. Let C_* be the chain complex with
$$C_n = \begin{cases} \mathbb{Z} & \text{if } n = 1 \\ 0 & \text{otherwise} \end{cases}$$
Let D_* be the chain complex with
$$D_n = \begin{cases} \mathbb{Z} & \text{if } n = 0, 1 \\ 0 & \text{otherwise} \end{cases}$$
such that the boundary map $\partial \colon D_1 \to D_0$ sends m to $2m$.

   Show that the natural projection $\pi \colon D_* \to C_*$ is a map of chain complexes and it induces the zero map $H_*(D_*) \to H_*(C_*)$.

2. With notations as in Question 1, show that there is no chain homotopy h such that $\partial h + h\partial = \pi$ (from π to zero).

3. For $Z \subset Y \subset X$ spaces, show that there is a short exact sequence of singular chain complexes
$$0 \to C_*(Y, Z) \to C_*(X, Z) \to C_*(X, Y) \to 0$$
What does the resulting long exact sequence of homology groups look like?

4. Hatcher, Exercise 12 on page 132.