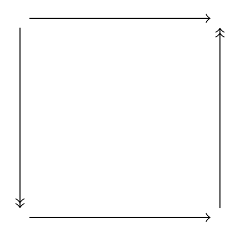
# MAT8021, Algebraic Topology

Assignment 8

Due in-class on Tuesday, April 13

1. Let $X$ be a Klein bottle:



   We can put a $\Delta$-complex structure on $X$ with one vertex $p$, three edges $a$, $b$, $c$, and two 2-simplices $u$, $v$. Make this $\Delta$-complex structure explicit, and use it to compute $H^*(X;\mathbb{Z}/2)$ together with the cup product on it.

2. In Hatcher, page 131, exercise 8, there is given a description of a *lens space* formed by gluing together $n$ tetrahedra; let's call this $L(n,1)$. (The 1 is because we are gluing the "bottom" face of $T_i$ to the top face of $T_{i+1}$.) Compute $H^*\big(L(n,1);\mathbb{Z}/n\big)$ together with the cup product on it.

3. We know that if $X$ and $Y$ are based spaces, the wedge $X \vee Y$ has

$$H^k(X \vee Y; R) = H^k(X;R) \oplus H^k(Y;R)$$

   for any $k > 0$. Show that under this identification, the cup product is given by

$$(\alpha, \beta) \smile (\alpha', \beta') = (\alpha \smile \alpha', \beta smile \beta')$$