MAT8021, Algebraic Topology

Assignment 7

Due in-class on Tuesday, April 6

1. Find all $(2,3)$-shuffles α and give formulas for the associated shuffle maps $f_\alpha \colon \Delta[5] \to \Delta[2] \times \Delta[3]$.

2. Find recursive formulas for $\dim_{\mathbb{Z}/2} H_k\big((\mathbb{RP}^2)^n; \mathbb{Z}/2\big)$ in terms of k and n.

3. Find a pair of chain complexes C_* and D_* such that the tensor product chain complex $C_* \otimes D_*$ does not satisfy the Künneth formula, i.e., there is some n such that

$$H_n(C_* \otimes D_*) \ncong \bigoplus_{p+q=n} H_p(C_*) \otimes H_q(D_*) \; \oplus \bigoplus_{p+q=n-1} \operatorname{Tor}\big(H_p(C_*), H_q(D_*)\big)$$

4. Suppose G is a topological group and X is a topological space with a continuous map $G \times X \to X$ which is an action of G. Show that $H_*(X)$ becomes a left module over the Pontrjagin ring $H_*(G)$.

5. Find the homology of the complex Grassmannian $\mathrm{Gr}_{\mathbb{C}}(3,5)$.

6. There is a continuous map from one Grassmannian $\mathrm{Gr}(k,n)$ to the next $\mathrm{Gr}(k,n+1)$ by sending a plane $V \subset \mathbb{R}^n$ to the plane

$$\{(0, x_1, \ldots, x_n) \mid (x_1, \ldots, x_n) \in V\}$$

Show that the image consists of a union of Schubert cells, and find the dimension of the smallest cell not in the image.