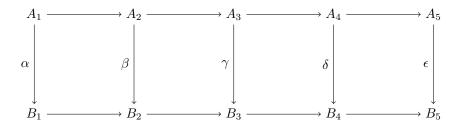
## MAT8021, Algebraic Topology

## Assignment 4

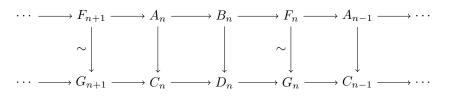
Due in-class on Tuesday, March 16

1. (The Five Lemma) Suppose



where the rows are exact and the squares commute. Suppose  $\alpha$ ,  $\beta$ ,  $\delta$ ,  $\epsilon$  are isomorphisms. Show that  $\gamma$  is an isomorphism.

- 2. Prove a stronger version of the Five Lemma: If  $\beta$  and  $\delta$  in the above diagram are injective, and  $\alpha$  is surjective, show that  $\gamma$  is injective. Give the dual statement (whose proof is of course essentially the same).
- 3. (Formal Mayer–Vietoris sequence) Suppose that there is a map of long exact sequence as follows:



Here all the maps  $F_n \to G_n$  are isomorphisms. Show that there is a long exact sequence

$$\cdots \to D_{n+1} \to A_n \to B_n \oplus C_n \to D_n \to A_{n-1} \to \cdots$$

(Define the maps first.)

4. Suppose X is a CW complex with finitely many cells. Define the Euler characteristic  $\chi(X)$  to be the number of even-dimensional cells minus the number of odd-dimensional cells.

If F is any field, show that

$$\chi(X) = \sum_{i} (-1)^{i} \dim_{F} \left( H_{i}(X;F) \right)$$

(Hint: Express this in terms of the ranks of the boundary maps.)