MAT8021, Algebraic Topology

Assignment 3

Due in-class on Tuesday, March 9

Numbered exercises are from Hatcher's "Algebraic Topology."

1. Let C_* be the chain complex with

$$C_n = \begin{cases} \mathbb{Z} & \text{if } n = 1 \\ 0 & \text{otherwise} \end{cases}$$

Let D_* be the chain complex with

$$D_n = \begin{cases} \mathbb{Z} & \text{if } n = 0, 1 \\ 0 & \text{otherwise} \end{cases}$$

such that the boundary map $\partial \colon D_1 \to D_0$ sends m to $2m$.

Show that the natural projection $\pi \colon D_* \to C_*$ is a map of chain complexes and it induces the zero map $H_*(D_*) \to H_*(C_*)$. Show that there is no chain homotopy h with $\partial h + h\partial = \pi$ (from π to zero).

2. For $Z \subset Y \subset X$ spaces, show that there is a short exact sequence of singular chain complexes

$$0 \to C_*(Y, Z) \to C_*(X, Z) \to C_*(X, Y) \to 0$$

What does the resulting long exact sequence of homology groups look like?

3. Hatcher, Exercise 12 on page 132.