

MAT8021, Algebraic Topology

Assignment 13

1. Show that if a principal bundle $P \rightarrow B$ has a section, then there is a homeomorphism to the trivial principal bundle: $P \cong B \times G$ as right G -spaces.
2. Let G and H be topological groups. Suppose $P_1 \rightarrow B$ is a principal G -bundle and $P_2 \rightarrow B$ is a principal H -bundle. Show that the pullback $P_1 \times_B P_2$ is a principal $G \times H$ -bundle.
3. Suppose $G \rightarrow H$ is a homomorphism of topological groups, and $P \rightarrow B$ is a principal G -bundle. Show that the mixing construction gives a *principal* H -bundle $P \times_G H \rightarrow B$.
4. We identified $\mathbb{C}\mathbb{P}^1$ with the space of lines in \mathbb{C}^2 . Associated to this, there is a vector bundle $\xi \rightarrow \mathbb{C}\mathbb{P}^1$:

$$\xi = \{(L, v) | L \in \mathbb{C}\mathbb{P}^1, v \in L\}$$

Find an open cover $\{U_\alpha\}$ together with transition functions $\{h_{\alpha,\beta}: U_\alpha \cap U_\beta \rightarrow \mathrm{GL}_1(\mathbb{C})\}$ to reconstruct the associated principal $\mathrm{GL}_1(\mathbb{C})$ -bundle.