

MAT7064, Topics in Geometry and Topology

Assignment 9

Due in-class on Friday, December 6

1. Show that the only possible nontrivial natural transformations $H^n(X; \mathbb{Z}) \rightarrow H^m(X; \mathbb{Q})$ occur when $m = nd$, and are of the form $\alpha \mapsto a\alpha^d$ for some $a \in \mathbb{Q}$.
2. Use rational cohomology to compute the rational homotopy groups $\pi_k(S^3 \vee S^3) \otimes \mathbb{Q}$ in dimensions $k = 1, \dots, 7$.
3. (This question is worth double.) A local coefficient system \mathcal{A} on a space X consists of the following data:
 - A set of abelian groups $\{A_x\}$ for $x \in X$.
 - For every path γ starting at x and ending at y , an isomorphism of abelian groups $\gamma_* : A_x \rightarrow A_y$ that only depends on the homotopy class of the path.

Recall that the standard n -simplex $\Delta[n] = \{(t_1, \dots, t_n) \mid 0 \leq t_1 \leq \dots \leq t_n \leq 1\}$. Given a local coefficient system \mathcal{A} on X , define the singular chain complex with values in \mathcal{A} by

$$C_n(X; \mathcal{A}) = \bigoplus_{\sigma: \Delta[n] \rightarrow X} A_{\sigma(1, \dots, 1)}$$

Use the structure of a local coefficient system to define a boundary map $\partial: C_n(X; \mathcal{A}) \rightarrow C_{n-1}(X; \mathcal{A})$ and show that it satisfies $\partial \circ \partial = 0$.

(The homology of the resulting chain complex is the *homology of X with coefficients in the coefficient system \mathcal{A}* . In particular, if $E \rightarrow B$ is a fibration, the homology groups of the fibers form local coefficient systems on B , and there is a version of the Serre spectral sequence that works with no assumptions on $\pi_1(B)$ acting trivially on $H_*(F)$.)