MAT7064, Topics in Geometry and Topology

Assignment 7

Due in-class on Friday, November 22

1. The special unitary group $SU(n)$ is the subgroup of $U(n)$ consisting of matrices of determinant 1, and there are fibration sequences

$$\begin{array}{ccc} SU(n) & \longrightarrow & U(n) \\ & & \downarrow \\ & & S^1 \end{array}$$

   where the last map is the determinant. Using this, the Serre spectral sequence, and the Hurewicz theorem, compute $\pi_k\big(U(n)\big)$ for $k \leq 3$.

2. Knowing $SO(3) \cong \mathbb{RP}^3$, compute $H^*\big((SO(4)\big)$ together with its cup product.

3. Using the Serre spectral sequence and the path-loop fibrations

$$\begin{array}{ccc} K(A,n) & \longrightarrow & * \\ & & \downarrow \\ & & K(A,n+1) \end{array}$$

   show that the rational cohomology groups $H^*\big(K(\mathbb{Z}/m,n);\mathbb{Q}\big)$ are trivial for all $m,n > 0$.

4. Compute the rational cohomology groups $H^*\big(K(\mathbb{Z},n);\mathbb{Q}\big)$ for all n.