MAT7064, Topics in Geometry and Topology

Assignment 6

Due in-class on Friday, November 8

1. Suppose $A \to X$ is a CW inclusion such that A is n-connected and the inclusion is m-connected. Find the strongest relationship that you can between $\pi_k(X/A)$ and $\pi_k(X,A)$ using the Blakers–Massey excision theorem.

2. Suppose G is an abelian group and $n \geq 2$. We know from algebra that we can find an exact sequence

$$0 \to R \to F \to G \to 0$$

where F is free on some set of generators $\{e_\alpha \mid \alpha \in A\} \subset F$ and R is free on some set of relations $\{f_\beta \mid \beta \in B\} \subset R$. Show that we can find a map

$$\bigvee_{\beta \in B} S^n \to \bigvee_{\alpha \in A} S^n$$

such that the induced map on π_n is isomorphic to the map $R \to F$.

3. (continuing the previous question) Show that we can construct a CW complex X, having cells only in dimensions n and $n+1$, with $\pi_k(X) = 0$ for $k < n$ and $\pi_n(X) = G$.

   Furthermore, show that we can construct a CW complex $K(G,n)$, having cells only in dimensions n and higher, with the only nonzero homotopy group being $\pi_n\big(K(G,n)\big) = G$.

4. (still continuing) Given any based space Y with $\pi_n(Y) = G$ and $\pi_k(Y) = 0$ for $k > n$, show that we can construct a map $K(G,n) \to Y$ which is an isomorphism in dimension n.

   Then use the Whitehead theorem to conclude that any two CW complexes with the only nonzero homotopy group, π_n, being isomorphic to G are homotopy equivalent.