MAT7064, Topics in Geometry and Topology

Assignment 4

Due in-class on Friday, October 18

1. Suppose that $i : A \to X$ is a cofibration, and let M_i be the mapping cylinder. Show that X/A is homotopy equivalent to the mapping cone $M_i/(A \times \{1\})$.

2. Suppose that $A \to X$ is a cofibration and $f : A \to Y$ is a map. We can form a new space $X \cup_A Y$ by gluing X to Y along A. Show that the map $Y \to X \cup_A Y$ is a cofibration.

3. Suppose X is a CW complex whose cells are of dimension d or less and Y is a space with $\pi_n(Y) = 0$ for $n \leq d$. Show that any map $X \to Y$ is null-homotopic.

4. A connected space Y that has only one nonzero homotopy group,

$$\pi_d(Y, y) = \begin{cases} G & \text{if } d = n, \\ 0 & \text{otherwise,} \end{cases}$$

is called an Eilenberg–Mac Lane space $K(G, n)$. Show that, for any CW complex X, the set $[X, K(G, n)]$ only depends on the quotient $X^{(n+1)}/X^{(n-2)}$ of the $(n + 1)$-skeleton by the $(n - 2)$-skeleton.