MAT7064, Topics in Geometry and Topology

Assignment 2

Due in-class on Friday, September 27

Numbered exercises are from Hatcher's "Algebraic Topology."

1. Suppose $X = S^1$ with basepoint $*$ and $A \subset S^1$ is a subspace (containing $*$) with exactly $k > 0$ points. Compute $\pi_n(X, A, *)$ for all $n \geq 1$.

2. Find an example of a pair of spaces $A \subset X$ with basepoint $*$ so that the map $\pi_1(X, *) \to \pi_1(X, A, *)$ cannot possibly be a group homomorphism.

3. Suppose X is a connected space and let $f : S^n \to X$ be any map. Show that f can be extended to a map $D^{n+1} \to X$ if and only if the image of f in $\pi_n\big(X, f(*)\big)$ is zero.

4. Suppose f is as in Question 3 and $g, h\colon D^{n+1} \to X$ are two extensions of f, i.e., $g|_{S^n} = h|_{S^n} = f$. Construct a "difference" $g - h \in \pi_{n+1}\big(X, f(*)\big)$ and show that there is a homotopy $H : D^{n+1} \times [0, 1] \to X$ from g to h that fixes the boundary S^n if and only if this difference is zero.