

# MAT7064, Topics in Geometry and Topology

## Assignment 1

Due in-class on Friday, September 20

Numbered exercises are from Hatcher's "Algebraic Topology."

1. Suppose  $A$  is a set with two binary operations  $\circ, *$  that share a common two-sided unit  $e$ : for any  $a \in A$ ,

$$a = e \circ a = a \circ e = e * a = a * e$$

Suppose that these two operations additionally satisfy an interchange law

$$(a * b) \circ (c * d) = (a \circ c) * (b \circ d)$$

Show that this implies  $a * b = a \circ b$  for any  $a, b \in A$ , and additionally  $a \circ b = b \circ a$ .

2. Suppose  $X$  is a space with a multiplication operation  $*$ :  $X \times X \rightarrow X$  having unit  $x_0 \in X$ , and let  $A = \pi_1(X, x_0)$ . Show that path composition  $\circ$  and pointwise multiplication  $(f * g)(t) = f(t) * g(t)$  satisfy the conditions of the previous problem, and hence the multiplication on  $\pi_1(X, x_0)$  is commutative.
3. Let  $B = [0, 1] \times [0, 1]$  be the unit square with boundary  $\partial B$ . Suppose  $X$  is a space with a chosen basepoint  $x_0$ , and let  $A$  be the set of maps  $f: B \rightarrow X$  such that  $f(\partial B) = \{x_0\}$ . By analogy with the fundamental group, define two "multiplication" operations  $\circ, *: A \times A \rightarrow A$  that satisfy the interchange rule listed in Question 1. (They have no unit until we pass to homotopy classes of maps.)
4. Hatcher, Exercise 2 on page 358.