MA341, Applied and Computational Topology

Assignment 3

Due in-class on Friday, December 5

1. Visualizing and interpreting patterns in high-dimensional data poses a significant challenge. Common approaches to address this issue include clustering, dimensionality reduction, and manifold learning. *Mapper* is an emerging TDA method, which is a combination of dimensionality reduction, clustering, and graph network techniques used to obtain a higher-level understanding of the structure of data. In this exercise, we use the Swiss Roll dataset as an example to explore the power of the Mapper algorithm in capturing and analyzing the structure of data. Follow the steps below and provide a report containing the visualization results.

   (a) Read relevant literature to understand the principles of the Mapper algorithm. As an accessible reference, here is an introduction to the algorithm from Sanjit Dandapanthula's notes.

   (b) Design your own Mapper algorithm to unfold the Swiss Roll and Swiss-Hole into a lower-dimensional space. Use `tda-mapper` or `KeplerMapper` to implement your algorithm. You may refer to this example for guidance. The results produced by your algorithm should be comparable to those obtained using the Locally Linear Embedding method in the example.

   Hint: Use the geodesic distance as the lens function in your Mapper algorithm.

   (c) Compare the results with those obtained from traditional dimensionality reduction techniques (PCA, UMAP, or t-SNE).

2. Give a concise but concrete description of your plan for the final project/report, specifically:

   (a) what real-world application of computational topology you would like to experiment on,

   (b) what you have done so far in terms of preparation,

   (c) the next steps you have in mind, and

   (d) one or two main references.

   Your final report is due by the end of the final exam weeks, that is, a month plus a week from now.