MA341, Applied and Computational Topology

Assignment 1

Due in-class on Friday, October 24

Numbered exercises are from Edelsbrunner and Harer's "Computational topology: An introduction."

1. Recall that the cube graph Q_3 is the graph formed by the 8 vertices and 12 edges of a 3-dimensional cube. We saw in class that it is a planar graph, i.e., embeddable to $\mathbb{R}^2$. More generally, embedding of a graph to a surface can be defined in a similar way. The embedding is said to be *regular*, if it possesses the greatest possible symmetry, like a regular polyhedron bound to the surface. For example, the planar embedding of Q_3 does not give rise to a regular one to S^2 via one-point compactification, but the latter does receive a regular embedding of Q_3 (one of the five *Platonic solids*).[1]

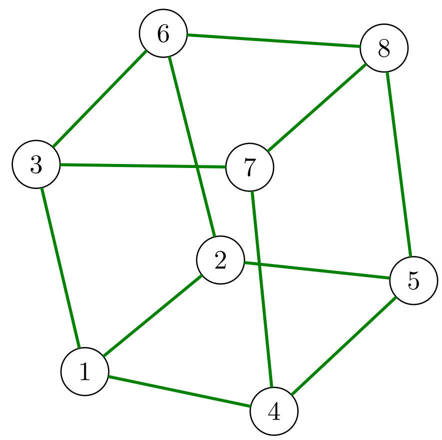
   Describe a regular embedding of Q_3 to the torus $S^1 \times S^1$. Hint: It may be convenient to illustrate the surface by a suitable parallelogram.

2. A *cubic graph* is a graph in which all vertices have degree 3, such as Q_3 above. Using the free open-source mathematics software system *SageMath*, list all planar cubic graphs with 8 vertices, up to isomorphism. What about allowing multigraphs? Hint: The *SageMath* website offers an extensive toolbox with numerous functions and examples for graph theory, among other subjects. The one you will need to enumerate planar graphs requires the additional package *plantri*, which is not available in the cloud version, but comes together with the local version downloadable to your computer.

   You are encouraged to explore what *SageMath* can do computationally with graphs (and knots and links). It can even render LaTeX code for the

[1] More precisely, an embedding M of a graph G is said to be regular if and only if for every two flags, i.e., triples (v_1, e_1, f_1) and (v_2, e_2, f_2), where e_i is an edge incident with the vertex v_i and the face f_i, there exists an automorphism of M which sends v_1 to v_2, e_1 to e_2, and f_1 to f_2.

outputs, such as this one:



3. Edelsbrunner–Harer, Exercise 1 on page 24.
4. Edelsbrunner–Harer, Exercise 5 on page 24.
5. Edelsbrunner–Harer, Exercise 7 on page 49.