Presentation Topics 1

1. Let A, B, C be sentences. Show that $\big(A \Rightarrow (B \vee C)\big) \Leftrightarrow \big((A \wedge \neg B) \Rightarrow C\big)$ is a tautology. Find another expression that is also equivalent to these.

2. Let A be a set. Show that $\emptyset \subseteq A$ and that $A \subseteq A$. If B is another set, show or disprove that $A \subseteq B$ or $B \subseteq A$.

3. Let A, B, C be sets. Show that if $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$. What is the analogous statement for four sets? What about N sets?

4. Let A, B, C be sets. Show that $C \subseteq A \cap B \iff C \subseteq A \wedge C \subseteq B$. Is there an analogous statement for $C \subseteq A \cup B$?

5. Let A be a set and let $\{B_\lambda\}_{\lambda \in \Lambda}$ be a collection of sets indexed by a set Λ. Show that $A \cap \left(\bigcup_{\lambda \in \Lambda} B_\lambda\right) = \bigcup_{\lambda \in \Lambda} (A \cap B_\lambda)$. Find and prove the analogous statement for $A \cap \left(\bigcap_{\lambda \in \Lambda} B_\lambda\right)$.

6. Let A, B be sets. Show that the following three conditions are equivalent:

   (a) $A \subseteq B$.

   (b) $A \cap B = A$.

   (c) $A \cup B = B$.

   Does the same hold if you reverse the roles of A and B?

7. Let A, B be subsets of a set U. Show that $(A \cap B)^c = A^c \cup B^c$. Does the same hold if you reverse the roles of the intersection and the union?

8. Let $\{A_\lambda\}_{\lambda \in \Lambda}$ be a collection of subsets of a set U indexed by a set Λ. Show that $\left(\bigcup_{\lambda \in \Lambda} A_\lambda\right)^c = \bigcap_{\lambda \in \Lambda} A_\lambda^c$. Does the same hold if you reverse the roles of the intersection and the union?

9. Let A, B, C be sets. Show that $C \setminus (A \cup B) = (C \setminus A) \cap (C \setminus B)$. What is the analogous statement for $C \setminus (A \cap B)$?

10. Let A, B, C be sets. Show that $(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B)$. What is the analogous statement for three sets?

11. Let A, B be sets. Show that $A \subseteq B \iff P(A) \subseteq P(B)$. Does the same hold if the containments are strict?

12. Let A, B be sets. Show that $P(A \cap B) = P(A) \cap P(B)$. Does the same hold if we take unions instead of intersections?

13. Let A, B be sets. Show that $P(A) \cup P(B) \subseteq P(A \cup B)$. Find an example to show that the containment can be strict.

14. Let A, B be sets and let $f \colon A \to B$ be a function. Assume that $X, Y \subseteq A$. Show that $f(X \cap Y) \subseteq f(X) \cap f(Y)$. Does the same hold if we take unions instead of intersections?

15. Let A, B be sets and let $f \colon A \to B$ be a function. Let Λ be an indexing set for a collection $\{T_\lambda\}_{\lambda \in \Lambda}$ of subsets of A. Show that $f(\bigcup_{\lambda \in \Lambda} T_\lambda) = \bigcup_{\lambda \in \Lambda} f(T_\lambda)$. Does the same hold if we take intersections instead of unions?

16. Let A, B be sets and let $f \colon A \to B$ be an invertible function. If $W, Z \subseteq B$, show that $f^{-1}(W \cap Z) = f^{-1}(W) \cap f^{-1}(Z)$. Does the same hold if we take unions instead of intersections?

17. Let A, B be sets and let $f \colon A \to B$ be a function. Let Λ be an indexing set for a collection $\{T_\lambda\}_{\lambda \in \Lambda}$ of subsets of B. Show that $f^{-1}(\bigcup_{\lambda \in \Lambda} T_\lambda) = \bigcup_{\lambda \in \Lambda} f^{-1}(T_\lambda)$. Does the same hold if we take intersections instead of unions?