(1) (a) $x+y$ is even
(b) If $x+y$ is odd, then either x is even or y is even.
(c) There exist x and y, Both x and y are odd, but $x+y$ is not even.
$\neg(P \Rightarrow Q) \iff P \wedge \neg Q$
(Note that there is an implicit universal quantifier in the original statement.)
(d) $\big((\exists m \in \mathbb{Z} \ni x = 2m+1) \wedge (\exists n \in \mathbb{Z} \ni y = 2n+1)\big) \implies (\exists k \in \mathbb{Z} \ni x+y = 2k)$.

(2) (a) Not decidable
(b) True
(c) False
(d) False

(3) (a) The number 3 is an element of A, so the set $\{3\}$ consisting of this single element is a subset of A. Therefore $\{3\} \in \mathcal{P}(A)$.
The number 13 is not in A, so $\{13\} \notin \mathcal{P}(A)$.
(b) There are 32 such elements.
Given any subset X of A, for each $a \in A$, the answer to "whether a lies in X" is YES or NO. Therefore, as a runs through all elements in A, we have $2 \times 2 \times 2 \times 2 \times 2 \times 2$ possible combinations of answers, each combination corresponding to a distinct subset X.
Now suppose $3 \notin X$. The answer to "3 lies X" must then be NO for all such X. Thus the possible combinations reduce to $2 \times 2 \times 1 \times 2 \times 2 \times 2 = 32$.

(4) (a) For any $b \in B$, there exists $a \in A$ such that $f(a) = b$.
(b) Let $A = \{1, 2\}$ and $B = \{3\}$. Define $f(1) = f(2) = 3$.

(5) (a) True
Proof To show $\overline{\overline{A} \cup \overline{B}} \subseteq A \cap B$, let $x \in \overline{\overline{A} \cup \overline{B}}$. Then $x \notin \overline{A} \cup \overline{B}$, and thus $x \notin \overline{A}$ and $x \notin \overline{B}$. In other words, $x \in A$ and $x \in B$. Therefore $x \in A \cap B$.

On the other hand, given any $y \in A \cap B$, we have $y \in A$ and $y \in B$, that is, $y \notin \overline{A}$ and $y \notin \overline{B}$. Therefore $y \notin \overline{A} \cup \overline{B}$, which means $y \in \overline{\overline{A} \cup \overline{B}}$. Hence $A \cap B \subseteq \overline{\overline{A} \cup \overline{B}}$. $\square$

(b) True
Proof By definition, $f^{-1}(B) = \{a \in A \mid f(a) \in B\}$. Since B is the codomain of f, $f(a) \in B$ for all a. Hence $f^{-1}(B) = A$. $\square$

(c) False
Let $A = \{1\}$ and $B = \{2, 3\}$. Define $f(1) = 2$. Consider $X = \{1\} \subseteq A$. Then $A \setminus X = \emptyset$ and so $f(A \setminus X) = \emptyset$. On the other hand, $f(X) = \{2\}$ so that $B \setminus f(X) = \{3\}$, which is not empty.