Math 300 MIDTERM NAME: ____________________________

Show **ALL** *work and write all proofs in full sentences. Answers without work and/or explanations will receive no credit. If a problem seems to be asking you to re-prove something from a lecture or the book, go through the proof rather than just quoting the result. If in doubt, ask.*

1. (10 points) For each of the following statements,

(i) rewrite the statement without words, using symbols such as $\forall$ and $\exists$, and

(ii) negate the statement.

(a) For all $x \in \mathbb{R}$, there exists $y \in \mathbb{R}$ such that $y^2 = x$.

(b) There exists $y \in \mathbb{R}$ such that $y^2 = x$ for all $x \in \mathbb{R}$.

2. (15 points) Consider the following implication (assume that x and y are integers).

If xy is odd then x is odd and y is odd.

(a) Write the converse of the implication.

(b) Write the negation of the implication.

(c) Write the contrapositive of the implication.

3. (9 points) Let $A = \{a, b, c, d\}$ and $B = \{b, c, e, f, g\}$. Write down an expression for each of the following sets in terms of the sets A and B using set operations (union, intersection, complement, etc.).

(a) $\{a, b, c, d, e, f, g\}$

(b) $\{e, f, g\}$

(c) $\{a, d, e, f, g\}$

4. (24 points) Let $\mathcal{P}(X)$ denote the power set of a set X.

(a) Let $A = \{1, 2, 3, 4, 5, 6\}$. For each of the following statements, write whether it is true or false.

- $\{3, 4\} \in \mathcal{P}(A)$
- $\{\{3, 4\}\} \in \mathcal{P}(A)$
- $\{\{3, 4\}\} \subseteq \mathcal{P}(A)$
- $\{\{3\}, \{4\}\} \subseteq \mathcal{P}(A)$

(b)

- What are the elements of $\mathcal{P}(\emptyset)$?
- What are the elements of $\mathcal{P}(\mathcal{P}(\emptyset))$?
- List all the subsets of $\mathcal{P}(\mathcal{P}(\emptyset))$.

5. (18 points) The following statements are both false. Prove this by giving a counterexample for each.

(a) Let U be a universal set. If A and B are sets, then $\overline{A} \cup \overline{B} = \overline{A \cup B}$.

(b) Let $f\colon X \to Y$ be a function. If A and B are subsets of X, then

$$f(A \cap B) = f(A) \cap f(B).$$

6. (12 points) Prove that if A and B are sets, then

$$A \subseteq B \Longleftrightarrow A \cap \overline{B} = \emptyset.$$

7. (12 points) Let $f\colon X \to Y$ be a function and $B \subseteq Y$. Prove that $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$.