Math 240 Quiz 7 (5.5,6.1,6.2)

NetID: ______________ **Class time:** ______________

Instructions: Calculators, course notes and textbooks are **NOT** allowed on the quiz. All numerical answers **MUST** be exact; e.g., you should write π instead of 3.14..., $\sqrt{2}$ instead of 1.414..., and $\frac{1}{3}$ instead of 0.3333... Explain your reasoning using complete sentences and correct grammar, spelling, and punctuation.
Show ALL of your work!
You have 20 minutes.

Question 1 (3 points). Find a matrix C of the form $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ and an invertible matrix P, both with real entries, such that

$$PCP^{-1} = \begin{bmatrix} 1 & 8 \\ -2 & 1 \end{bmatrix}$$

$$\det \begin{bmatrix} 1-\lambda & 8 \\ -2 & 1-\lambda \end{bmatrix} = (1-\lambda)^2 + 16 = 0 \implies \lambda = 1 \pm 4i$$

we find eigenvector for $\lambda = 1-4i$

$$\begin{bmatrix} 1-(1-4i) & 8 \\ -2 & 1-(1-4i) \end{bmatrix} \sim \begin{bmatrix} i & 2 \\ 1 & -2i \end{bmatrix} \sim \begin{bmatrix} 1 & -2i \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 4i & 8 \\ -2 & 4i \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \implies \begin{array}{l} x_1 - 2i\,x_2 = 0 \\ x_1 = 2i\,x_2 \end{array}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 2i \\ 1 \end{bmatrix}$$ and $\begin{bmatrix} 2i \\ 1 \end{bmatrix}$ is an eigenvector for $1-4i$

therefore $C = \begin{bmatrix} 1 & -4 \\ 4 & 1 \end{bmatrix}$ and $P = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$

check

$$PC = \begin{bmatrix} 8 & 2 \\ 1 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 8 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$$

Question 2 (3points). Find a basis for $W^{\perp}$ where W is

$$W = \text{Span}\{\begin{bmatrix}1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\1\end{bmatrix}\}$$

Let $A = \begin{bmatrix}1 & 1 & 0 & 0\\0 & 0 & 1 & 1\end{bmatrix}$, Then $(\text{Row } A)^{\perp} = \text{Nul } A$

$\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}$ in Nul A $\rightarrow$ $\begin{bmatrix}1 & 1 & 0 & 0\\0 & 0 & 1 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = 0$

$x_1 + x_2 = 0 \rightarrow x_1 = -x_2$

$x_3 + x_4 = 0 \quad\quad x_3 = -x_4$

$$\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = x_2\begin{bmatrix}-1\\1\\0\\0\end{bmatrix} + x_4\begin{bmatrix}0\\0\\-1\\1\end{bmatrix}$$

$$\text{Nul } A = \text{span}\left\{\begin{bmatrix}-1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\-1\\1\end{bmatrix}\right\}$$

Question 3 (4 point). Mark each statement True, False. Justify your answer.

i. Let U and V be $n \times n$ orthogonal matrices. Then UV is invertible. True

$U^T U = Id$ $\Rightarrow (UV)^T (UV) = V^T U^T U V = Id$ $\Rightarrow (UV)^{-1} = (UV)^T$

$V^T V = Id$

ii. A square matrix with orthonormal rows is an orthogonal matrix. True

$UU^T = Id \Rightarrow U^{-1} = U^T$ and so orthogonal

iii. Let $u = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ and $y = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$. Let $L = \text{Span}\{u\}$, then the distance from u to L is 4. False

$\text{proj}_L y = \frac{y \cdot u}{\|u\|^2} u = \frac{30}{40} u = \frac{3}{4} \begin{bmatrix} 2 \\ 6 \end{bmatrix}$

$\text{dist}(y, L) = \| y - \text{proj}_L y \| = \left\| \begin{bmatrix} 3 \\ 4 \end{bmatrix} - \begin{bmatrix} \frac{3}{2} \\ \frac{9}{2} \end{bmatrix} \right\| = \sqrt{\frac{10}{4}}$

iv. The matrix $\begin{bmatrix} 2 & -3 \\ 3 & 2 \end{bmatrix}$ preserve the length of any vector in $\mathbb{R}^2$.

False $\begin{bmatrix} 2 & -3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$\left\| \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\| = 1$

$\left\| \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\| = \sqrt{13}$