Math 240 Quiz 5 (3.1-3.2)

NetID: ______________ **Class time:** ______________

Instructions: Calculators, course notes and textbooks are **NOT** allowed on the quiz. All numerical answers **MUST** be exact; e.g., you should write π instead of 3.14..., $\sqrt{2}$ instead of 1.414..., and $\frac{1}{3}$ instead of 0.3333... Explain your reasoning using complete sentences and correct grammar, spelling, and punctuation.
Show ALL of your work!
You have 20 minutes.

Question 1 (6 points). Compute the determinant of the following matrix.

$$A = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 4 & 4 & 7 & 4 \\ 0 & 2 & 3 & 2 \\ 0 & 0 & 8 & 3 \end{bmatrix}$$

Are the column vectors of A linearly independent?

Expand det A along the first column:

$$\det A = 1 \cdot \begin{vmatrix} 4 & 7 & 4 \\ 2 & 3 & 2 \\ 0 & 8 & 3 \end{vmatrix} - 4 \cdot \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 0 & 8 & 3 \end{vmatrix}$$

$$= \left(4 \cdot \begin{vmatrix} 3 & 2 \\ 8 & 3 \end{vmatrix} - 2 \cdot \begin{vmatrix} 7 & 4 \\ 8 & 3 \end{vmatrix} \right) - 4 \left(1 \cdot \begin{vmatrix} 3 & 2 \\ 8 & 3 \end{vmatrix} - 2 \cdot \begin{vmatrix} 2 & 1 \\ 8 & 3 \end{vmatrix} \right)$$

$$= 4(-7) - 2(-11) - 4(-7+4)$$

$$= 6$$

Since $\det A \neq 0$, the columns are linearly independent.

Question 2 (4 points). Consider the matrix

$$M = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 3 \\ 1 & 0 & 3 \end{bmatrix}$$

(i) Determine the values of h such that $\det(M - hI) = 0$.

$$\det(M-hI) = \det\begin{bmatrix} 1-h & 0 & -1 \\ 0 & 2-h & 3 \\ 1 & 0 & 3-h \end{bmatrix}$$

$$= (1-h)(2-h)(3-h) + (2-h)$$

$$= (2-h)(4-4h+h^2)$$

$$= (2-h)^3$$

Thus h can only be 2.

(ii) Determine the values of λ such that $\det(M^2 - \lambda I) = 0$.

$$M^2 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 3 \\ 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 3 \\ 1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -4 \\ 3 & 4 & 15 \\ 4 & 0 & 8 \end{bmatrix}$$

$$\det(M^2-\lambda I) = \det\begin{bmatrix} -\lambda & 0 & -4 \\ 3 & 4-\lambda & 15 \\ 4 & 0 & 8-\lambda \end{bmatrix}$$

$$= (4-\lambda)\begin{vmatrix} -\lambda & -4 \\ 4 & 8-\lambda \end{vmatrix}$$

$$= (4-\lambda)(\lambda^2 - 8\lambda + 16)$$

$$= (4-\lambda)^3$$

Therefore $\lambda = 4$.