Math 240 Quiz 4 (2.1-2.3)

NetID: ____________ **Class time:** ____________

Instructions: Calculators, course notes and textbooks are **NOT** allowed on the quiz. All numerical answers **MUST** be exact; e.g., you should write π instead of 3.14..., $\sqrt{2}$ instead of 1.414..., and $\frac{1}{3}$ instead of 0.3333... Explain your reasoning using complete sentences and correct grammar, spelling, and punctuation.
Show ALL of your work!
You have 20 minutes.

Question 1 (5 points)**.** Let

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

i. Find the inverse of A if it exists.

$$A = \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 2 & 1 & -1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{array}\right]$$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

ii. Find the solution set of the linear system of equations $A\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

$$\mathbf{x} = A^{-1}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}.$$

Question 2 (5 points)**.** Decide which of the following statements are true, and justify your answer.

i. $A = \begin{bmatrix} 3 & 4 \\ 3 & 0 \end{bmatrix}$ and B and C are 2×3 matrices such that $AB = AC$. Then $B = C$.

True. $\det(A) \neq 0$ and so A is invertible. Thus

$$B = A^{-1}AB = A^{-1}AC = C.$$

ii. The matrix $A = \begin{bmatrix} 2 & 2 & -1 \\ -2 & 4 & 1 \\ -8 & -3 & 4 \end{bmatrix}$ is invertible.

False. $\begin{bmatrix} 2 \\ -2 \\ -8 \end{bmatrix} = -2 \begin{bmatrix} -1 \\ 1 \\ 4 \end{bmatrix}$. Therefore the columns of A are not linearly independent. So A is not invertible.

iii. Let T be a linear transformation from $\mathbb{R}^n$ into $\mathbb{R}^n$. Suppose $T(\mathbf{u}) = T(\mathbf{v})$ for some $\mathbf{u} \neq \mathbf{v}$ in $\mathbb{R}^n$. Then T cannot be onto.

True. $T(\mathbf{u}) = T(\mathbf{v})$ therefore $T(\mathbf{u} - \mathbf{v}) = 0$ and $\mathbf{u} - \mathbf{v} \neq 0$. If A is the standard matrix for T, then the system $A\mathbf{x} = 0$ has non-trivial solution. So A is not invertible and so does T. Therefore T is not onto.

iv. The linear transformation obtained by rotation of angle θ with respect to the origin, $T(\mathbf{x}) = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \mathbf{x}$, is one-to-one.

True. $\det(\begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}) = \cos^2(\theta) + \sin^2(\theta) = 1 \neq 0$. Therefore T is invertible and so is one-to-one.

v. Let $\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$. Then $\mathbf{x}^T\mathbf{x} = \mathbf{x}\mathbf{x}^T$.

False. $\mathbf{x}^T\mathbf{x}$ is a 3×3 matrix where $\mathbf{x}\mathbf{x}^T$ is a 1×1 matrix.