Math 240 Quiz 1 (1.1-1.3)

NetID: ______________ **Class time:** ______________

Instructions: Calculators, course notes and textbooks are **NOT** allowed on the quiz. All numerical answers **MUST** be exact; e.g., you should write π instead of 3.14..., $\sqrt{2}$ instead of 1.414..., and $\frac{1}{3}$ instead of 0.3333... Explain your reasoning using complete sentences and correct grammar, spelling, and punctuation.
Show ALL of your work!
You have 20 minutes.

Question 1 (2 points). Solve the system

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & x_3 & = & 3 \\ -2x_1 & + & 4x_2 & + & 5x_3 & = & -5 \\ 3x_1 & - & 6x_2 & - & 6x_3 & = & 8 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 3 \\ -2 & 4 & 5 & -5 \\ 3 & -6 & -6 & 8 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & -1 & 3 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & -3 & -1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & -1 & 3 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\Rightarrow \begin{cases} x_1 = 2x_2 + x_3 + 3 = 2x_2 + \frac{10}{3} \\ x_2 \text{ is free} \\ x_3 = \frac{1}{3} \end{cases}$$

Question 2 (2 points). Use row reduction to put the following matrix into reduced echelon form:

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Question 3 (3 points). Determine the value(s) of h and k such that the system has (a) no solution, (b) a unique solution.

$$\begin{aligned} x_1 &- 3x_2 &= h \\ 5x_1 &+ kx_2 &= -7 \end{aligned}$$

$$\left[\begin{array}{cc|c} 1 & -3 & h \\ 5 & k & -7 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -3 & h \\ 0 & 15+k & -5h-7 \end{array}\right]$$

(a) The system has no solution if

$15+k=0$ and $-5h-7 \neq 0$

that is, $k=-15$ and $h \neq -\frac{7}{5}$.

(b) It has a unique solution if $15+k \neq 0$,

that is, $k \neq -15$ and h arbitrary

Question 4 (3 points). Let

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}, \quad \mathbf{a}_2 = \begin{bmatrix} -2 \\ -3 \\ 7 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ 17 \end{bmatrix}.$$

Determine if $\mathbf{b}$ is a linear combination of $\mathbf{a}_1$ and $\mathbf{a}_2$.

$$\left[\begin{array}{cc|c} 1 & -2 & 4 \\ 4 & -3 & 1 \\ 2 & 7 & 17 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 5 & -15 \\ 0 & 11 & 9 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 11 & 9 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & 42 \end{array}\right]$$

This system has no solution.

Thus $\vec{b}$ is not a linear combination of $\vec{a}_1$ and $\vec{a}_2$.