Math 240 Midterm Exam - 2015

Name:________________________

Instructions: Calculators, course notes and textbooks are **NOT** allowed. All numerical answers **MUST** be exact; e.g., you should write π instead of 3.14..., $\sqrt{2}$ instead of 1.414..., and $\frac{1}{3}$ instead of 0.3333... Explain your reasoning using complete sentences and correct grammar, spelling, and punctuation.
Show ALL of your work!

Score	
1	
2	
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Total	

Question 1 (15 points)**.** Row reduce the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 0 & 1 \\ 3 & -2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

to reduced echelon form. Find the null space and the column space of A by writing down a basis for each.

Question 2 (10 points)**.** Find all solutions to the following:

$$\begin{aligned} x_1 - x_2 + x_3 &= 3 \\ 2x_2 - x_3 &= 2 \\ 3x_1 - x_2 + 2x_3 &= 11 \end{aligned}$$

Question 3 (10 points)**.** Find the inverse of the following matrix:

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 5 & 2 \\ 1 & 0 & 4 \end{bmatrix}.$$

Question 4 (10 points)**.** Let

$$\mathbf{a}_1 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} 4 \\ 2 \\ -2 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$$

Is the vector $\mathbf{b}$ in the span of $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$?

Question 5 (15 point)**.** Find all solutions of the matrix equation $A\mathbf{x} = \mathbf{b}$ in parametric form, where

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 2 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}.$$

Question 6 (15 points)**.** True or false? Justify your answers.

(a) A nonhomogeneous equation with at most one free variable has at most one solution.

(b) The equation $A\mathbf{x} = \mathbf{0}$ has at least one nontrivial solution.

(c) If AB is not invertible, then either A or B must not be invertible.

(d) If A is a 3×5 matrix then the linear transformation $T(\mathbf{x}) = A\mathbf{x}$ is onto.

(e) If the solution of $A\mathbf{x} = \mathbf{b}$ is unique for every $\mathbf{b}$, then the linear transformation $T(\mathbf{x}) = A\mathbf{x}$ is onto.

Question 7 (15 points)**.** Let the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ be given by

$$T(x_1, x_2, x_3) = (-x_1 + x_2 + x_3, x_2 + 3x_3).$$

(1) Find the standard matrix for T;
(2) Show that T is onto.
(3) Is T one-to-one? Justify your answer.

Question 8 (10 points)**.** Consider the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Assuming A is invertible, compute $\det A^{-1}$, the determinant of A^{-1}.